Complete Solutions Manual

Elementary Geometry for College Students

SEVENTH EDITION

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Chapter 1 Line and Angle Relationships

SECTION 1.1: Early Definitions and Postulates

1. *AC*

2. Midpoint

3. 6.25 ft · 12 in./ft = 75 in.

4. 52 in. ÷ 12 in./ft = $4\frac{1}{3}$ ft or 4 ft 4 in.

5. $\frac{1}{2}\text{m} \cdot 3.28 \text{ ft/m} = 1.64 \text{ feet}$

6. 16.4 ft ÷ 3.28 ft/m = 5 m

7. 18 – 15 = 3 mi

8. $300 + 450 + 600 = 1350$ ft
1350 ft ÷ 15 ft/s = 90 s or 1 min 30 s

9. **a.** *A-C-D*

b. *A*, *B*, *C* or *B*, *C*, *D* or *A*, *B*, *D*

10. **a.** Infinite

b. One

c. None

d. None

11. $\overleftrightarrow{CD}$ means line *CD*;
$\overline{CD}$ means segment *CD*;
CD means the measure or length of $\overline{CD}$;
$\overrightarrow{CD}$ means ray *CD* with endpoint *C*.

12. **a.** No difference

b. No difference

c. No difference

d. $\overrightarrow{CD}$ is the ray starting at *C* and going toward *D*.
$\overrightarrow{DC}$ is the ray starting at *D* and going toward *C*.

13. **a.** *m* and *t*

b. *m* and *p* or *p* and *t*

14. **a.** False

b. False

c. True

d. True

e. False

15.
$$\begin{aligned} 2x + 1 &= 3x - 2 \\ -x &= -3 \\ x &= 3 \\ AM &= 7 \end{aligned}$$

16.
$$\begin{aligned} 2(x+1) &= 3(x-2) \\ 2x + 2 &= 3x - 6 \\ -1x &= -8 \\ x &= 8 \\ AB &= AM + MB \\ AB &= 18 + 18 = 36 \end{aligned}$$

17.
$$\begin{aligned} 2x + 1 + 3x + 2 &= 6x - 4 \\ 5x + 3 &= 6x - 4 \\ -1x &= -7 \\ x &= 7 \\ AB &= 38 \end{aligned}$$

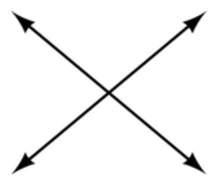
18. No; Yes; Yes; No

19. **a.** $\overrightarrow{OA}$ and $\overrightarrow{OD}$

b. $\overrightarrow{OA}$ and $\overrightarrow{OB}$
(There are other possible answers.)

20. $\overleftrightarrow{CD}$ lies on plane *X*.

21. **a.**



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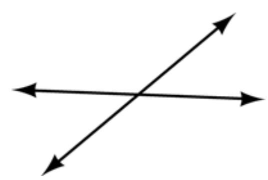

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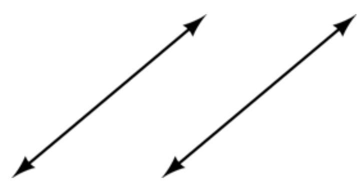
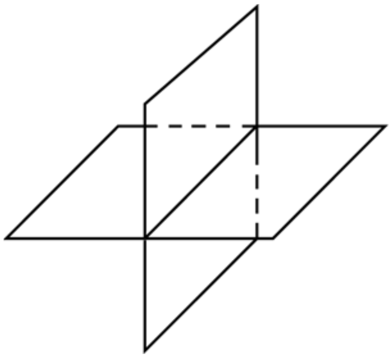


22. a.



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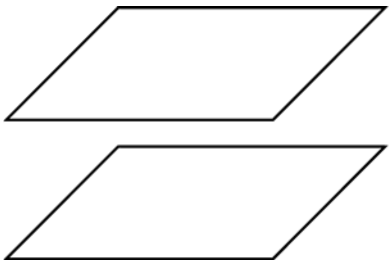

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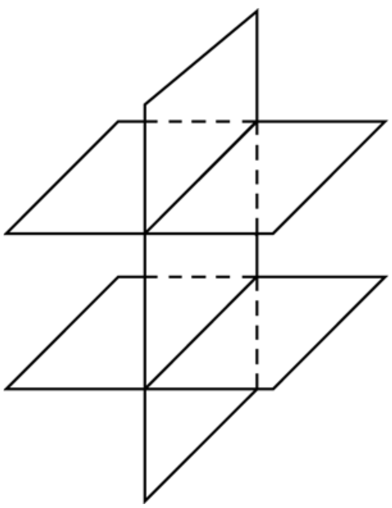


23. Planes M and N intersect at $\overleftrightarrow{AB}$.

24. *B*

25. *A*

26. a. One

b. Infinite

c. One

d. None

27. a. *C*

b. *C*

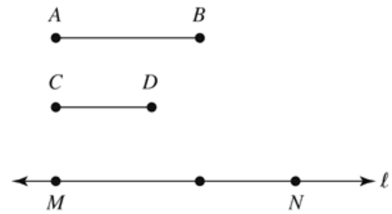
c. *H*

28. a. Equal

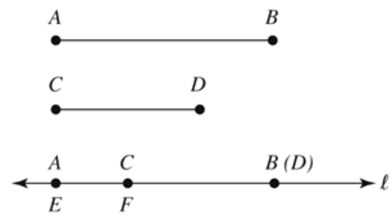
b. Equal

c. *AC* is twice *CD*.

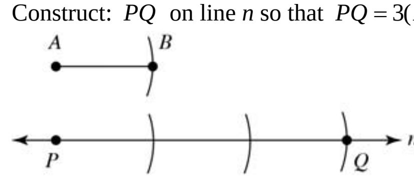
29. Given: $\overline{AB}$ and $\overline{CD}$ as shown ($AB > CD$)
Construct $\overline{MN}$ on line ℓ so that
$MN = AB + CD$



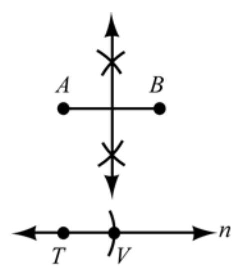
30. Given: $\overline{AB}$ and $\overline{CD}$ as shown ($AB > CD$)
Construct: $\overline{EF}$ on line ℓ so that $EF = AB - CD$.



31. Given: $\overline{AB}$ as shown
Construct: $\overline{PQ}$ on line n so that $PQ = 3(AB)$



32. Given: $\overline{AB}$ as shown
Construct: $\overline{TV}$ on line n so that $TV = \frac{1}{2}(AB)$



33. a. No

b. Yes

c. No

d. Yes

34. A segment can be divided into 2^n congruent parts, where $n \geq 1$.

35. Six

36. Four

37. Nothing

38. **a.** One

b. One

c. None

d. One

e. One

f. One

g. None

39. **a.** Yes

b. Yes

c. No

40. **a.** Yes

b. No

c. Yes

41. $\frac{1}{3}a + \frac{1}{2}b$ or $\frac{2a+3b}{6}$

SECTION 1.2: Angles and Their Relationships

1. **a.** Acute

b. Right

c. Obtuse

2. **a.** Obtuse

b. Straight

c. Acute

3. **a.** Complementary

b. Supplementary

4. **a.** Congruent

b. None

5. Adjacent

6. Vertical

7. Complementary (also adjacent)

8. Supplementary

9. Yes; No

10. **a.** True

b. False

c. False

d. False

e. True

11. **a.** Obtuse

b. Straight

c. Acute

d. Obtuse

12. B is not in the interior of $\angle FAE$; the Angle-Addition Postulate does not apply.

13. $m\angle FAC + m\angle CAD = 180$
$\angle FAC$ and $\angle CAD$ are supplementary.

14. **a.** $x + y = 180$

b. $x = y$

15. **a.** $x + y = 90$

b. $x = y$

16. 62°

17. 42°

18.
$$\begin{aligned} 2x + 9 + 3x - 2 &= 67 \\ 5x + 7 &= 67 \\ 5x &= 60 \\ x &= 12 \end{aligned}$$

19.
$$\begin{aligned} 2x - 10 + x + 6 &= 4(x - 6) \\ 3x - 4 &= 4x - 24 \\ 20 &= x \\ x &= 20 \\ m\angle RSV &= 4(20 - 6) = 56° \end{aligned}$$

20.
$$\begin{aligned} 5(x+1) - 3 + 4(x-2) + 3 &= 4(2x+3) - 7 \\ 5x + 5 - 3 + 4x - 8 + 3 &= 8x + 12 - 7 \\ 9x - 3 &= 8x + 5 \\ x &= 8 \\ m\angle RSV = 4(2 \cdot 8 + 3) - 7 &= 69° \end{aligned}$$

21. $\frac{x}{2} + \frac{x}{4} = 45$

Multiply by LCD, 4

$$\begin{aligned} 2x + x &= 180 \\ 3x &= 180 \\ x &= 60;\ m\angle RST = 30° \end{aligned}$$

22. $\frac{2x}{3}+\frac{x}{2}=49$

Multiply by LCD, 6

$$4x+3x=294$$
$$7x=294$$
$$x=42;\ m\angle TSV=\frac{x}{2}=21°$$

23. $$x+y=2x-2y$$
$$x+y+2x-2y=64$$

$$-1x+3y=0$$
$$3x-1y=64$$

$$-3x+9y=0$$
$$3x-y=64$$
$$8y=64$$
$$y=8;\ x=24$$

24. $$2x+3y=3x-y+2$$
$$2x+3y+3x-y+2=80$$

$$-1x+4y=2$$
$$5x+2y=78$$

$$-5x+20y=10$$
$$5x+2y=78$$
$$22y=88$$
$$y=4;\ x=14$$

25. $\angle CAB\cong\angle DAB$

26. $$x+y=90$$
$$x=12+y$$

$$x+y=90$$
$$x-y=12$$
$$2x=102$$
$$x=51$$
$$51+y=90$$
$$y=39$$
$\angle$s are 51° and 39°.

27. $$x+y=180$$
$$x=24+2y$$

$$x+y=180$$
$$x-2y=24$$

$$2x+2y=360$$
$$x-2y=24$$
$$3x=384$$
$$x=128;\ y=52$$
$\angle$s are 128° and 52°.

28. **a.** $(90-x)°$

b. $(90-(3x-12))°=(102-3x)°$

c. $(90-(2x+5y))°=(90-2x-5y)°$

29. **a.** $(180-x)°$

b. $(180-(3x-12))°=(192-3x)°$

c. $(180-(2x+5y))°=(180-2x-5y)°$

30. $$x-92=92-53$$
$$x-92=39$$
$$x=131$$

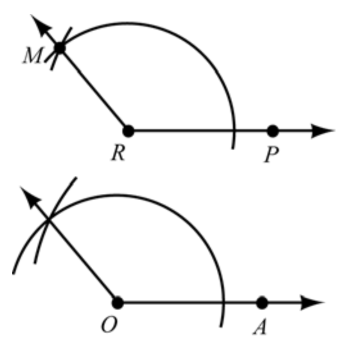
31. $$x-92+(92-53)=90$$
$$x-92+39=90$$
$$x-53=90$$
$$x=143$$

32. **a.** True

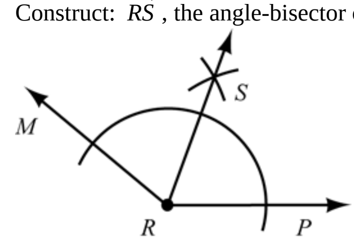
b. False

c. False

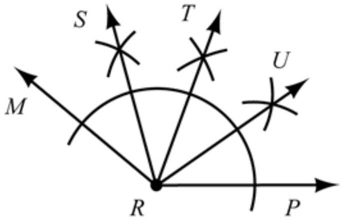
33. Given: Obtuse $\angle MRP$
Construct: With $\overrightarrow{OA}$ as one side, an angle $\cong\angle MRP$



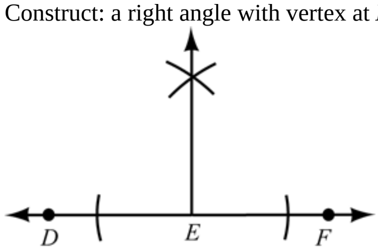
34. Given: Obtuse $\angle MRP$
Construct: $\overrightarrow{RS}$, the angle-bisector of $\angle MRP$



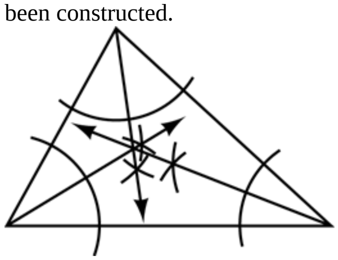
35. Given: Obtuse $\angle MRP$
Construct: Rays *RS*, *RT*, and *RU* so that $\angle MRP$ is divided into 4 $\cong$ angles



36. Given: Straight angle *DEF*
Construct: a right angle with vertex at *E*

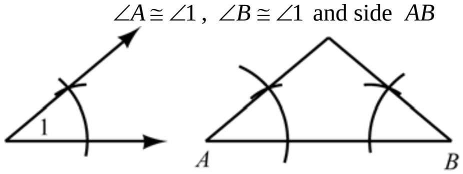


37. For the triangle shown, the angle bisectors have been constructed.



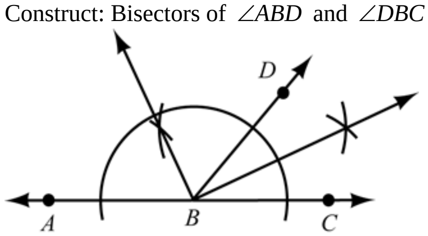
It appears that the angle bisectors meet at one point.

38. Given: Acute $\angle 1$ and $\overleftrightarrow{AB}$
Construct: Triangle *ABC* which has $\angle A \cong \angle 1$, $\angle B \cong \angle 1$ and side $\overline{AB}$



39. It appears that the two sides opposite $\angle$ s *A* and *B* are congruent.

40. Given: Straight $\angle ABC$ and $\overrightarrow{BD}$
Construct: Bisectors of $\angle ABD$ and $\angle DBC$



It appears that a right angle is formed.

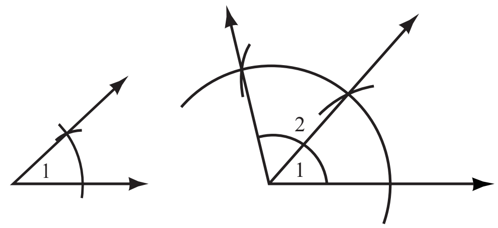
41. $m\angle 1 + m\angle 2 = 90°$

If $\angle$s 1 and 2 are bisected, then

$\frac{1}{2} \cdot m\angle 1 + \frac{1}{2} \cdot m\angle 2 = 45°$

42. Given: Acute $\angle 1$

Construct: $\angle 2$, an angle whose measure is twice that of $\angle 1$



43. **a.** 90°

b. 90°

c. Equal

44. Let $m\angle USV = x$, then $m\angle TSU = 38 - x$

$$38 - x + 40 = 61$$
$$78 - x = 61$$
$$78 - 61 = x$$

$$x = 17;\ m\angle USV = 17°$$

45. $x + 2z + x - z + 2x - z = 60$

$$4x = 60$$
$$x = 15$$

If $x = 15$, then $m\angle USV = 15 - z$,
$m\angle VSW = 2(15) - z$, and
$m\angle USW = 3x - 6 = 3(15) - 6 = 39$

So $15 - z + 2(15) - z = 39$

$$45 - 2z = 39$$
$$6 = 2z$$
$$z = 3$$

46. **a.** 52°

b. 52°

c. Equal

47. $90 + x + x = 360$

$$2x = 270$$
$$x = 135°$$

48. 90°

SECTION 1.3: Introduction to Geometric Proof

1. Division Property of Equality or Multiplication Property of Equality
2. Distributive Property $[x+x=(1+1)x=2x]$
3. Subtraction Property of Equality
4. Addition Property of Equality
5. Multiplication Property of Equality
6. Addition Property of Equality
7. If 2 angles are supplementary, then the sum of their measures is 180°.
8. If the sum of the measures of 2 angles is 180°, then the angles are supplementary.
9. Angle-Addition Property
10. Definition of angle-bisector
11. $AM+MB=AB$
12. $AM=MB$
13. $\overrightarrow{EG}$ bisects $\angle DEF$
14. $m\angle 1=m\angle 2$ or $\angle 1\cong\angle 2$
15. $m\angle 1+m\angle 2=90^\circ$
16. $\angle 1$ and $\angle 2$ are complementary
17. $2x=10$
18. $x=7$
19. $7x+2=30$
20. $\frac{1}{2}=50\%$
21. $6x-3=27$
22. $x=-20$
23. **1.** Given
    2. Distributive Property
    3. Addition Property of Equality
    4. Division Property of Equality
24. **1.** Given
    2. Subtraction Property of Equality
    3. Division Property of Equality
25. **1.** $2(x+3)-7=11$
    2. $2x+6-7=11$
    3. $2x-1=11$
    4. $2x=12$
    5. $x=6$
26. **1.** $\frac{x}{5}+3=9$
    2. $\frac{x}{5}=6$
    3. $x=30$
27. **1.** Given
    2. Segment-Addition Postulate
    3. Subtraction Property of Equality
28. **1.** Given
    2. The midpoint forms 2 segments of equal measure.
    3. Segment-Addition Postulate
    4. Substitution
    5. Distributive Property
    6. Multiplication (or Division) Property of Equality
29. **1.** Given
    2. If an angle is bisected, then the two angles formed are equal in measure.
    3. Angle-Addition Postulate
    4. Substitution
    5. Distribution Property
    6. Multiplication (or Division) Property of Equality
30. **1.** Given
    2. Angle-Addition Postulate
    3. Subtraction Property of Equality
31. **S1.** M-N-P-Q on $\overline{MQ}$
    R1. Given
    2. Segment-Addition Postulate
    3. Segment-Addition Postulate
    4. $MN+NP+PQ=MQ$
32. **S1.** $\angle TSW$ with $\overrightarrow{SU}$ and $\overrightarrow{SV}$
    R1. Given
    2. Angle-Addition Postulate
    3. Angle-Addition Postulate
    4. $m\angle TSW=m\angle TSU+m\angle USV+m\angle VSW$

33. $5 \cdot x + 5 \cdot y = 5(x + y)$

34. $5 \cdot x + 7 \cdot x = (5 + 7)x = 12x$

35. $(-7)(-2) > 5(-2)$ or $14 > -10$

36. $\frac{12}{-4} < \frac{-4}{-4}$ or $-3 < 1$

37. $ac > bc$

38. $x > -5$

39. **1**. Given

2. Addition Property of Equality

3. Given

4. Substitution

40.

1. $a = b$	**1**. Given
2. $a - c = b - c$	**2**. Subtraction Property of Equality
3. $c = d$	**3**. Given
4. $a - c = b - d$	**4**. Substitution

SECTION 1.4: Relationships: Perpendicular Lines

1. **1.** Given

2. If 2 $\angle$ s are $\cong$, then they are equal in measure.

3. Angle-Addition Postulate

4. Addition Property of Equality

5. Substitution

6. If 2 $\angle$ s are = in measure, then they are $\cong$.

2. **1.** Given

2. The measure of a straight angle is 180°.

3. Angle-Addition Postulate

4. Substitution

5. Given

6. The measure of a right $\angle = 90^\circ$.

7. Substitution

8. Subtraction Property of Equality

9. Angle-Addition Postulate

10. Substitution

11. If the sum of measures of 2 angles is 90°, then the angles are complementary.

3. **1.** $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$

2. $\angle 1 \cong \angle 3$

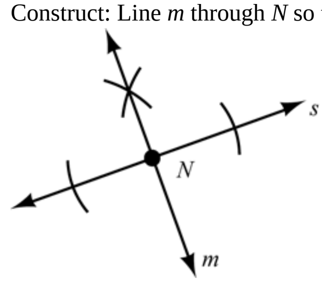
4. **1.** $m\angle AOB = m\angle 1$ and $m\angle BOC = m\angle 1$

2. $m\angle AOB = m\angle BOC$

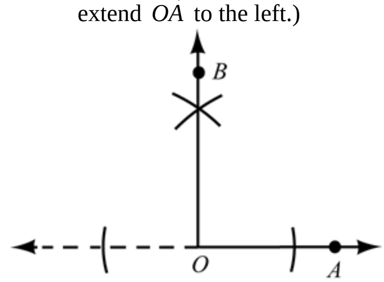
3. $\angle AOB \cong \angle BOC$

4. $\overrightarrow{OB}$ bisects $\angle AOC$

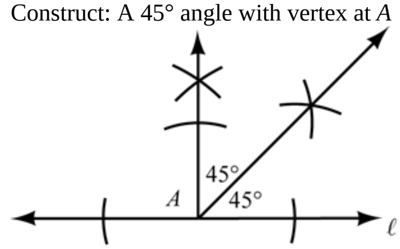
5. Given: Point *N* on line *s*.
Construct: Line *m* through *N* so that $m \perp s$



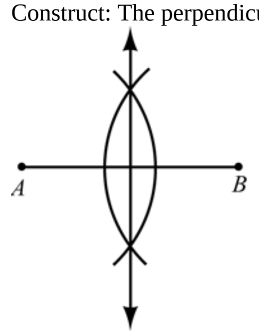
6. Given: $\overrightarrow{OA}$
Construct: Right angle *BOA*
(Hint: Use the straightedge to extend $\overrightarrow{OA}$ to the left.)



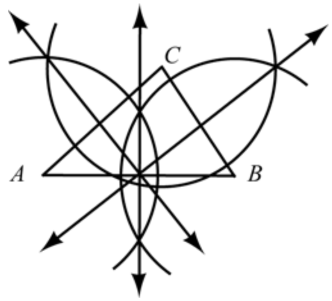
7. Given: Line ℓ containing point *A*
Construct: A 45° angle with vertex at *A*



8. Given: $\overline{AB}$
Construct: The perpendicular bisector of $\overline{AB}$



9. Given: Triangle ABC
Construct: The perpendicular bisectors of sides, $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$



10. It appears that the perpendicular bisectors meet at one point.

11. **R1.** Given

R3. Substitution

S4. $m\angle 1 = m\angle 2$

S5. $\angle 1 \cong \angle 2$

12. **R1.** Given

S2. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$

R3. Given

S4. $m\angle 2 + m\angle 3 = 90$

R5. Substitution

S6. $\angle$s 1 and 4 are complementary.

13. No; Yes; No

14. No; No; Yes

15. No; Yes; No

16. No; No; Yes

17. No; Yes; Yes

18. No; No; No

19. **a.** perpendicular

b. angles

c. supplementary

d. right

e. measure of angle

20. **a.** postulate

b. union

c. empty set

d. less than

e. point

21. **a.** adjacent

b. complementary

c. ray AB

d. is congruent to

e. vertical

22. In space, there is an infinite number of lines perpendicular to a given line at a point on the line.

23.

STATEMENTS	REASONS
1. M-N-P-Q on $\overline{MQ}$	**1.** Given
2. $MN + NQ = MQ$	**2.** Segment-Addition Postulate
3. $NP + PQ = NQ$	**3.** Segment-Addition Postulate
4. $MN + NP + PQ = MQ$	**4.** Substitution

24. $AE = AB + BC + CD + DE$

25.

STATEMENTS	REASONS
1. $\angle TSW$ with $\overrightarrow{SU}$ and $\overrightarrow{SV}$	**1.** Given
2. $m\angle TSW = m\angle TSU + m\angle USW$	**2.** Angle-Addition Postulate
3. $m\angle USW = m\angle USV + m\angle VSW$	**3.** Angle-Addition Postulate
4. $m\angle TSW = m\angle TSU + m\angle USV + m\angle VSW$	**4.** Substitution

26. $m\angle GHK = m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4$

27. In space, there is an infinite number of lines that perpendicularly bisect a given line segment at its midpoint.

28. **1.** Given

2. If 2 $\angle$s are complementary, then the sum of their measures is 90°.

3. Given

4. The measure of an acute angle is between 0 and 90°.

5. Substitution

6. Subtraction Property of Equality

7. Subtraction Property of Inequality

8. Addition Property of Inequality

9. Transitive Property of Inequality

10. Substitution

11. If the measure of an angle is between 0 and 90°, then the angle is an acute $\angle$.

29. Angles 1, 2, 3, and 4 are adjacent and form the straight angle *AOB*, which measures 180. Therefore, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180$.

30. If $\angle 2$ and $\angle 3$ are complementary, then $m\angle 2 + m\angle 3 = 90$. From Exercise 29, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180$. Therefore, $m\angle 1 + m\angle 4 = 90$ and $\angle 1$ and $\angle 4$ are complementary.

SECTION 1.5: The Formal Proof of a Theorem

1. H: A line segment is bisected.

C: Each of the equal segments has half the length of the original segment.

2. H: Two sides of a triangle are congruent.

C: The triangle is isosceles.

3. First write the statement in the "If, then" form. If a figure is a square, then it is a quadrilateral.

H: A figure is a square.

C: It is a quadrilateral.

4. First write the statement in the "If, then" form. If a polygon is a regular polygon, then it has congruent interior angles.

H: A polygon is a regular polygon.

C: It has congruent interior angles.

5. First write the statement in the "If, then" form. If each is right angle, then two angles are congruent.

H: Each is a right angle.

C: Two angles are congruent.

6. First write the statement in the "If, then" form. If polygons are similar, then the lengths of corresponding sides are proportional.

H: Polygons are similar.

C: The lengths of corresponding sides are proportional.

7. Statement, Drawing, Given, Prove, Proof

8. **a.** Hypothesis

b. Hypothesis

c. Conclusion

9. **a**. Given **b**. Prove

10. *a*, *c*, *d*

11. After the theorem has been proved.

12. No

13. Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$
Prove: $\angle AEC$ is a right angle.

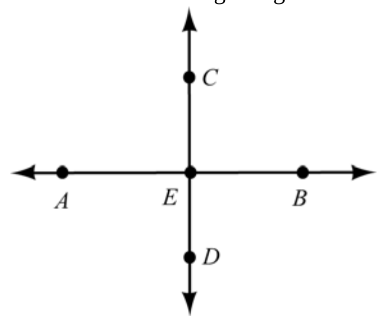
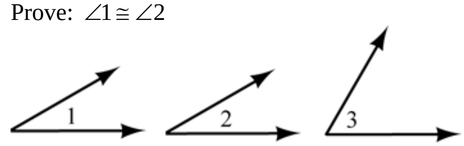


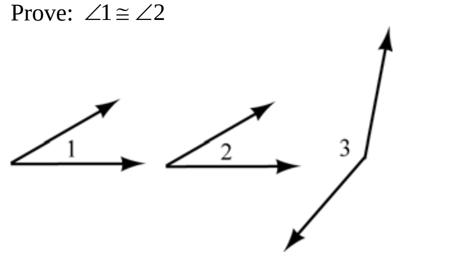
Figure for exercises 13 and 14.

14. Given: $\angle AEC$ is a right angle
Prove: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

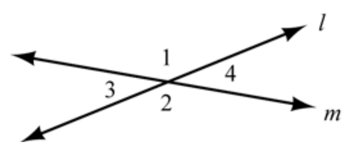
15. Given: $\angle 1$ is complementry to $\angle 3$
$\angle 2$ is complementry to $\angle 3$
Prove: $\angle 1 \cong \angle 2$



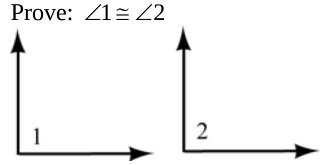
16. Given: $\angle 1$ is supplementary to $\angle 3$
$\angle 2$ is supplementary to $\angle 3$
Prove: $\angle 1 \cong \angle 2$



17. Given: Lines *l* and *m* intersect as shown
Prove: $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$



18. Given: $\angle 1$ and $\angle 2$ are right angles
Prove: $\angle 1 \cong \angle 2$



19. $m\angle 2 = 55^\circ$, $m\angle 3 = 125^\circ$, $m\angle 4 = 55^\circ$

20. $m\angle 1 = 133^\circ$, $m\angle 3 = 133^\circ$, $m\angle 4 = 47^\circ$

21.
$$m\angle 1 = m\angle 3$$
$$3x + 10 = 4x - 30$$
$$x = 40;\ m\angle 1 = 130^\circ$$

22. $m\angle 2 = m\angle 4$
$6x + 8 = 7x$
$x = 8;\ m\angle 2 = 56°$

23. $m\angle 1 + m\angle 2 = 180°$
$2x + x = 180$
$3x = 180$
$x = 60;\ m\angle 1 = 120$

24. $m\angle 2 + m\angle 3 = 180°$
$x + 15 + 2x = 180$
$3x = 165$
$x = 55;\ m\angle 2 = 70°$

25. $\frac{x}{2} - 10 + \frac{x}{3} + 40 = 180$
$\frac{x}{2} + \frac{x}{3} + 30 = 180$
$\frac{x}{2} + \frac{x}{3} = 150$

Multiply by 6

$3x + 2x = 900$
$5x = 900$
$x = 180;\ m\angle 2 = 80°$

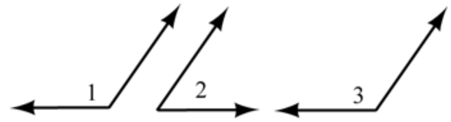
26. $x + 20 + \frac{x}{3} = 180$
$x + \frac{x}{3} = 160$

Multiply by 3

$3x + x = 480$
$4x = 480$
$x = 120;\ m\angle 4 = 40°$

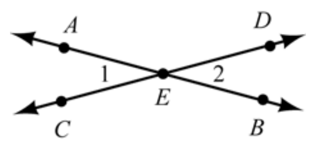
27.
1. Given
2. If 2 $\angle$ s are complementary, the sum of their measures is 90.
3. Substitution
4. Subtraction Property of Equality
5. If 2 $\angle$ s are = in measure, then they are $\cong$.

28. Given: $\angle 1$ is supplementary to $\angle 2$
$\angle 3$ is supplementary to $\angle 2$
Prove: $\angle 1 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ is supplementary to $\angle 2$ $\angle 3$ is supplementary to $\angle 2$	**1.** Given
2. $m\angle 1 + m\angle 2 = 180$ $m\angle 3 + m\angle 2 = 180$	**2.** If 2 $\angle$s are supplementary, then the sum of their measures is 180.
3. $m\angle 1 + m\angle 2$ $= m\angle 3 + m\angle 2$	**3.** Substitution
4. $m\angle 1 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	**5.** If 2 $\angle$s are = in measure, then they are $\cong$.

29. If 2 lines intersect, the vertical angles formed are congruent.
Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E
Prove: $\angle 1 \cong \angle 2$



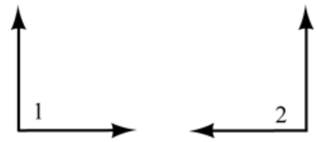
STATEMENTS	REASONS
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E	**1.** Given
2. $\angle 1$ is supplementary to $\angle AED$ $\angle 2$ is supplementary to $\angle AED$	**2.** If the exterior sides of two adjacent $\angle$s form a straight line, then these $\angle$s are supplementary
3. $\angle 1 \cong \angle 2$	**3.** If 2 $\angle$s are supplementary to the same $\angle$, then these $\angle$s are $\cong$.

30. Any two right angles are congruent.

Given: $\angle 1$ is a right $\angle$

$\angle 2$ is a right $\angle$

Prove: $\angle 1 \cong \angle 2$



STATEMENTS	REASONS
1. $\angle 1$ is a right $\angle$ $\angle 2$ is a right $\angle$	**1.** Given
2. $m\angle 1 = 90$ $m\angle 2 = 90$	**2.** Measure of a right $\angle = 90$.
3. $m\angle 1 = m\angle 2$	**3.** Substitution
4. $\angle 1 \cong \angle 2$	**4.** If 2 $\angle$s are = in measure, then they are $\cong$.

31. **R1.** Given

S2. $\angle ABC$ is a right $\angle$.

R3. The measure of a right $\angle = 90$.

R4. Angle-Addition Postulate

S6. $\angle 1$ is complementary to $\angle 2$.

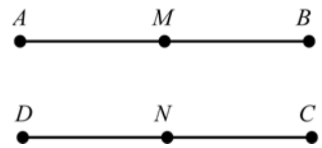
32. If 2 segments are congruent, then their midpoints separate these segments into four congruent segments.

Given: $\overline{AB} \cong \overline{DC}$

M is the midpoint of $\overline{AB}$

N is the midpoint of $\overline{DC}$

Prove: $\overline{AM} \cong \overline{MB} \cong \overline{DN} \cong \overline{NC}$



STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DC}$	**1.** Given
2. $AB = DC$	**2.** If 2 segments are $\cong$, then their lengths are $=$.
3. $AB = AM + MB$ $DC = DN + NC$	**3.** Segment-Addition Postulate
4. $AM + MB = DN + NC$	**4.** Substitution
5. M is the midpoint of $\overline{AB}$ N is the midpoint of $\overline{DC}$	**5.** Given
6. $AM = MB$ and $DN = NC$	**6.** If a point is the midpoint of a segment, it forms 2 segments equal in measure.
7. $AM + AM = DN + DN$ or $2 \cdot AM = 2 \cdot DN$	**7.** Substitution
8. $AM = DN$	**8.** Division Property of Equality
9. $AM = MB = DN = NC$	**9.** Substitution
10. $\overline{AM} \cong \overline{MB} \cong \overline{DN} \cong \overline{NC}$	**10.** If segments are = in length, then they are $\cong$.

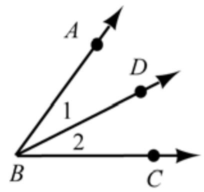
33. If 2 angles are congruent, then their bisectors separate these angles into four congruent angles.

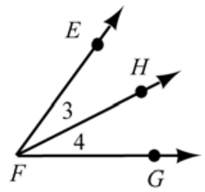
Given: $\angle ABC \cong \angle EFG$

$\overrightarrow{BD}$ bisects $\angle ABC$

$\overrightarrow{FH}$ bisects $\angle EFG$

Prove: $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$





STATEMENTS	REASONS
1. $\angle ABC \cong \angle EFG$	**1.** Given
2. $m\angle ABC = m\angle EFG$	**2.** If 2 angles are $\cong$, their measures are $=$.
3. $m\angle ABC = m\angle 1 + m\angle 2$ $m\angle EFG = m\angle 3 + m\angle 4$	**3.** Angle-Addition Postulate
4. $m\angle 1 + m\angle 2$ $= m\angle 3 + m\angle 4$	**4.** Substitution
5. $\overrightarrow{BD}$ bisects $\angle ABC$ $\overrightarrow{FH}$ bisects $\angle EFG$	**5.** Given
6. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$	**6.** If a ray bisects an $\angle$, then 2 $\angle$s of equal measure are formed.
7. $m\angle 1 + m\angle 1$ $= m\angle 3 + m\angle 3$ or $2 \cdot m\angle 1 = 2 \cdot m\angle 3$	**7.** Substitution
8. $m\angle 1 = m\angle 3$	**8.** Division Property of Equality
9. $m\angle 1 = m\angle 2$ $= m\angle 3 = m\angle 4$	**9.** Substitution
10. $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$	**10.** If $\angle$s are = in measure, then they are $\cong$.

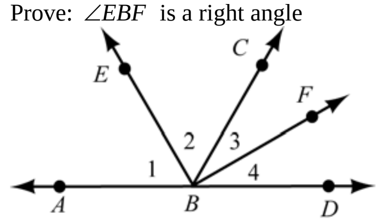
34. The bisectors of two adjacent supplementary angles form a right angle.

Given: $\angle ABC$ is supplementary to $\angle CBD$

$\overrightarrow{BE}$ bisects $\angle ABC$

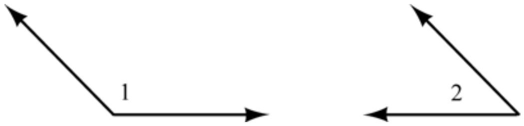
$\overrightarrow{BF}$ bisects $\angle CBD$

Prove: $\angle EBF$ is a right angle



STATEMENTS	REASONS
1. $\angle ABC$ is supplementary to $\angle CBD$	**1.** Given
2. $m\angle ABC + m\angle CBD$ $= 180$	**2.** The sum of the measures of supplementary angles is 180.
3. $m\angle ABC = m\angle 1 + m\angle 2$ $m\angle CBD = m\angle 3 + m\angle 4$	**3.** Angle-Addition Postulate
4. $m\angle 1 + m\angle 2 + m\angle 3$ $+ m\angle 4 = 180$	**4.** Substitution
5. $\overrightarrow{BE}$ bisects $\angle ABC$ $\overrightarrow{BF}$ bisects $\angle CBD$	**5.** Given
6. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$	**6.** If a ray bisects an $\angle$, then 2 $\angle$s of equal measure are formed.
7. $m\angle 2 + m\angle 2 + m\angle 3$ $+ m\angle 3 = 180$ or $2 \cdot m\angle 2 + 2 \cdot m\angle 3 = 180$	**7.** Substitution
8. $m\angle 2 + m\angle 3 = 90$	**8.** Division Property of Equality
9. $m\angle EBF = m\angle 2 + m\angle 3$	**9.** Angle-Addition Postulate
10. $m\angle EBF = 90$	**10.** Substitution
11. $\angle EBF$ is a right angle	**11.** If the measure of an $\angle$ is 90, then the $\angle$ is a right $\angle$.

35. The supplement of an acute angle is obtuse.
Given: $\angle 1$ is supplementary to $\angle 2$
$\angle 2$ is an acute $\angle$
Prove: $\angle 1$ is an obtuse $\angle$



	STATEMENTS		REASONS
1.	$\angle 1$ is supplementary to $\angle 2$	**1.**	Given
2.	$m\angle 1 + m\angle 2 = 180$	**2.**	If 2 $\angle$s are supplementary, the sum of their measures is 180.
3.	$\angle 2$ is an acute $\angle$	**3.**	Given
4.	$m\angle 2 = x$ where $0 < x < 90$	**4.**	The measure of an acute $\angle$ is between 0 and 90.
5.	$m\angle 1 + x = 180$	**5.**	Substitution (#4 into #2)
6.	x is positive $\therefore m\angle 1 < 180$	**6.**	If $a + p_1 = b$ and p_1 is positive, then $a < b$.
7.	$m\angle 1 = 180 - x$	**7.**	Substitution Property of Equality (#5)
8.	$-x < 0 < 90 - x$	**8.**	Subtraction Property of Inequality (#4)
9.	$90 - x < 90 < 180 - x$	**9.**	Addition Property of Inequality (#8)
10.	$90 - x < 90 < m\angle 1$	**10.**	Substitution (#7 into #9)
11.	$90 < m\angle 1 < 180$	**11.**	Transitive Property of Inequality (#6 & #10)
12.	$\angle 1$ is an obtuse $\angle$	**12.**	If the measure of an angle is between 90 and 180, then the $\angle$ is obtuse.

CHAPTER REVIEW

1. Undefined terms, defined terms, axioms or postulates, theorems

2. Induction, deduction, intuition

3. **1.** Names the term being defined.

2. Places the term into a set or category.

3. Distinguishes the term from other terms in the same category.

4. Reversible

4. Intuition

5. Induction

6. Deduction

7. H: The diagonals of a trapezoid are equal in length.

C: The trapezoid is isosceles.

8. H: The parallelogram is a rectangle.

C: The diagonals of a parallelogram are congruent.

9. No conclusion

10. Jody Smithers has a college degree.

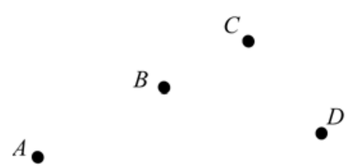
11. Angle A is a right angle.

12. C

13. $\angle RST$ or $\angle TSR$, $\angle S$, greater than 90°.

14. Diagonals are $\perp$ and they bisect each other.

15.



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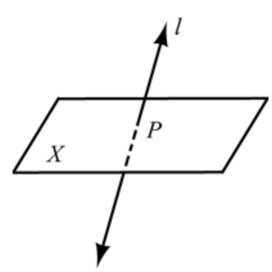

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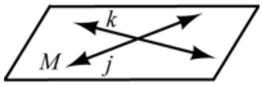


18. **a.** Obtuse **b.** Right

19. **a.** Acute **b.** Reflex

20.
$$\begin{aligned} 2x+15 &= 3x+5 \\ 10 &= x \\ x = 10;\ m\angle ABC &= 70^\circ \end{aligned}$$

21.
$$\begin{aligned} 2x+5+3x-4 &= 86 \\ 5x+1 &= 86 \\ 5x &= 85 \\ x = 17;\ m\angle DBC &= 47^\circ \end{aligned}$$

22.
$$\begin{aligned} 3x-1 &= 4x-5 \\ 4 &= x \\ x = 4;\ AB &= 22 \end{aligned}$$

23.
$$\begin{aligned} 4x-4+5x+2 &= 25 \\ 9x-2 &= 25 \\ 9x &= 27 \\ x = 3;\ MB &= 17 \end{aligned}$$

24.
$$\begin{aligned} 2\cdot CD &= BC \\ 2(2x+5) &= x+28 \\ 4x+10 &= x+28 \\ 3x &= 18 \\ x = 6;\ AC = BC &= 6+28 = 34 \end{aligned}$$

25.
$$\begin{aligned} 7x-21 &= 3x+7 \\ 4x &= 28 \\ x &= 7 \end{aligned}$$
$$m\angle 3 = 49-21 = 28^\circ$$
$$\therefore m\angle FMH = 180-28 = 152^\circ$$

26.
$$\begin{aligned} 4x+1+x+4 &= 180 \\ 5x+5 &= 180 \\ 5x &= 175 \\ x &= 35 \end{aligned}$$
$$m\angle 4 = 35+4 = 39^\circ$$

27. **a.** Point M

b. $\angle JMH$

c. $\overrightarrow{MJ}$

d. $\overleftrightarrow{KH}$

28.
$$\begin{aligned} 2x-6+3(2x-6) &= 90 \\ 2x-6+6x-18 &= 90 \\ 8x-24 &= 90 \\ 8x &= 114 \\ x &= 14\tfrac{1}{4} \end{aligned}$$
$$\begin{aligned} m\angle EFH &= 3(2x-6) = 3\left(28\tfrac{1}{2}-6\right) \\ &= 3\cdot 22\tfrac{1}{2} \\ &= 67\tfrac{1}{2}^\circ \end{aligned}$$

29.
$$\begin{aligned} x+(40+4x) &= 180 \\ 5x+40 &= 180 \\ 5x &= 140 \\ x &= 28^\circ \\ 40+4x &= 152^\circ \end{aligned}$$

30. **a.** $2x+3+3x-2+x+7 = 6x+8$

b.
$$\begin{aligned} 6x+8 &= 32 \\ 6x &= 24 \\ x &= 4 \end{aligned}$$

c.
$$\begin{aligned} 2x+3 &= 2(4)+3 = 11 \\ 3x-2 &= 3(4)-2 = 10 \\ x+7 &= 4+7 = 11 \end{aligned}$$

31. The measure of angle 3 is less than 50°.

32. The four foot board is 48 inches. Subtract 6 inches on each end, leaving 36 inches.
$$\begin{aligned} 4(n-1) &= 36 \\ 4n-4 &= 36 \\ 4n &= 40 \\ n &= 10 \end{aligned}$$
$\therefore$ 10 pegs will fit on the board.

33. S

34. S

35. A

36. S

37. N

38. **S2.** $\angle 4 \cong \angle P$

S3. $\angle 1 \cong \angle 4$

R4. If 2 $\angle$ s are $\cong$, then their measures are =.

R5. Given

S6. $m\angle 2 = m\angle 3$

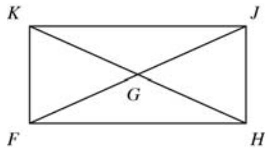
S7. $m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$

R8. Angle-Addition Postulate

R9. Substitution

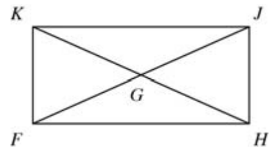
S10. $\angle TVP \cong \angle MVP$

39. Given: $\overline{KF} \perp \overline{FH}$
$\angle JHK$ is a right $\angle$
Prove: $\angle KFH \cong \angle JHF$



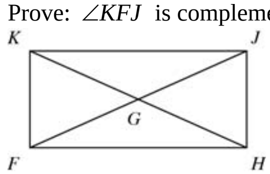
STATEMENTS	REASONS
1. $\overline{KF} \perp \overline{FH}$	**1.** Given
2. $\angle KFH$ is a right $\angle$	**2.** If two segments are $\perp$, then they form a right $\angle$.
3. $\angle JHF$ is a right $\angle$	**3.** Given
4. $\angle KFH \cong \angle JHF$	**4.** Any two right $\angle$s are $\cong$.

40. Given: $\overline{KH} \cong \overline{FJ}$
G is the midpoint of both $\overline{KH}$ and $\overline{FJ}$
Prove: $\overline{KG} \cong \overline{GJ}$



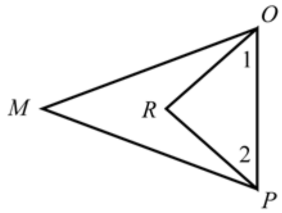
STATEMENTS	REASONS
1. $\overline{KH} \cong \overline{FJ}$ G is the midpoint of both $\overline{KH}$ and $\overline{FJ}$	**1.** Given
2. $\overline{KG} \cong \overline{GJ}$	**2.** If 2 segments are $\cong$, then their midpoints separate these segments into 4 $\cong$ segments.

41. Given: $\overline{KF} \perp \overline{FH}$
Prove: $\angle KFJ$ is complementary to $\angle JFH$



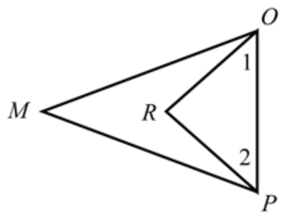
STATEMENTS	REASONS
1. $\overline{KF} \perp \overline{FH}$	**1.** Given
2. $\angle KFH$ is complementary to $\angle JFH$	**2.** If the exterior sides of 2 adjacent $\angle$s form $\perp$ rays, then these $\angle$s are complementary

42. Given: $\angle 1$ is complementary to $\angle M$
$\angle 2$ is complementary to $\angle M$
Prove: $\angle 1 \cong \angle 2$



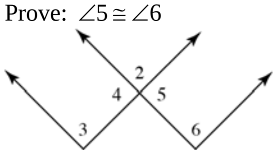
STATEMENTS	REASONS
1. $\angle 1$ is complementary to $\angle M$	**1.** Given
2. $\angle 2$ is complementary to $\angle M$	**2.** Given
3. $\angle 1 \cong \angle 2$	**3.** If 2 $\angle$s are complementary to the same $\angle$, then these angles are $\cong$.

43. Given: $\angle MOP \cong \angle MPO$
$\overrightarrow{OR}$ bisects $\angle MOP$
$\overrightarrow{PR}$ bisects $\angle MPO$
Prove: $\angle 1 \cong \angle 2$



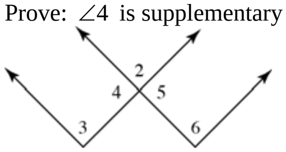
STATEMENTS	REASONS
1. $\angle MOP \cong \angle MPO$	**1.** Given
2. $\overrightarrow{OR}$ bisects $\angle MOP$ $\overrightarrow{PR}$ bisects $\angle MPO$	**2.** Given
3. $\angle 1 \cong \angle 2$	**3.** If 2 $\angle$s are $\cong$, then their bisectors separate these $\angle$s into 4 $\cong$ $\angle$s.

44. Given: $\angle 4 \cong \angle 6$
Prove: $\angle 5 \cong \angle 6$



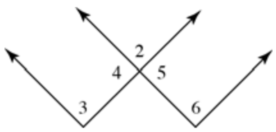
STATEMENTS	REASONS
1. $\angle 4 \cong \angle 6$	**1.** Given
2. $\angle 4 \cong \angle 5$	**2.** If 2 angles are vertical $\angle$s then they are $\cong$.
3. $\angle 5 \cong \angle 6$	**3.** Transitive Property

45. Given: Figure as shown
Prove: $\angle 4$ is supplementary to $\angle 2$



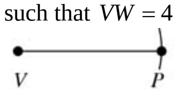
STATEMENTS	REASONS
1. Figure as shown	**1.** Given
2. $\angle 4$ is supplementary to $\angle 2$	**2.** If the exterior sides of 2 adjacent $\angle$s form a line, then the $\angle$s are supplementary

46. Given: $\angle 3$ is supplementary to $\angle 5$
$\angle 4$ is supplementary to $\angle 6$
Prove: $\angle 3 \cong \angle 6$

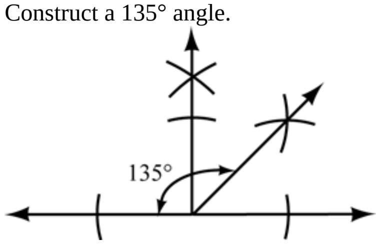


STATEMENTS	REASONS
1. $\angle 3$ is supplementary to $\angle 5$ $\angle 4$ is supplementary to $\angle 6$	**1.** Given
2. $\angle 4 \cong \angle 5$	**2.** If 2 lines intersect, the vertical angles formed are $\cong$.
3. $\angle 3 \cong \angle 6$	**3.** If 2 $\angle$s are supplementary to congruent angles, then these angles are $\cong$.

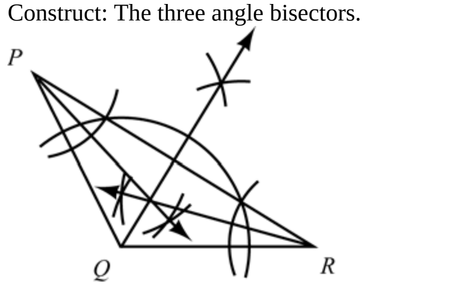
47. Given: $\overline{VP}$
Construct: $\overline{VW}$ such that $VW = 4 \cdot VP$



48. Construct a 135° angle.

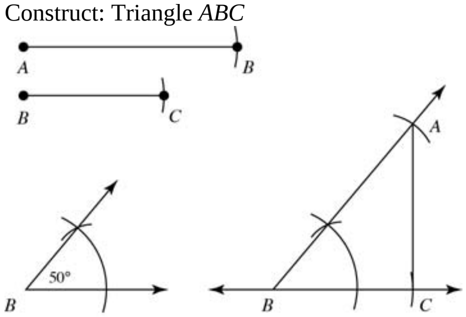


49. Given: Triangle PQR
Construct: The three angle bisectors.

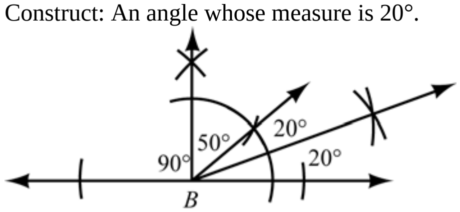


It appears that the three angle bisectors meet at one point inside the triangle.

50. Given: $\overline{AB}$, $\overline{BC}$, and $\angle B$ as shown
Construct: Triangle ABC



51. Given: $m\angle B = 50^\circ$
Construct: An angle whose measure is 20°.



52. $m\angle 2 = 270^\circ$

CHAPTER TEST

1. $\angle CBA$ or $\angle B$

2. $AP + PB = AB$

3. **a.** Point

b. Line

4. **a.** Right

b. Obtuse

5. **a.** Supplementary

b. Congruent

6. $m\angle MNP = m\angle PNQ$

7. **a.** Right

b. Supplementary

8. Right $\angle$

9. Addition Property of Equality

10. $3.2 + 7.2 = 10.4$ in.

11. **a.**
$$\begin{aligned} x + x + 5 &= 27 \\ 2x + 5 &= 27 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

b. $x + 5 = 11 + 5 = 16$

12. $m\angle 4 = 35^\circ$

13. **a.**
$$\begin{aligned} x + 2x - 3 &= 69 \\ 3x - 3 &= 69 \\ 3x &= 72 \\ x &= 24^\circ \end{aligned}$$

b. $m\angle 4 = 2(24) - 3 = 45^\circ$

14. **a.** $m\angle 2 = 137^\circ$

b. $m\angle 3 = 43^\circ$

15. **a.**
$$\begin{aligned} 2x - 3 &= 3x - 28 \\ x &= 25^\circ \end{aligned}$$

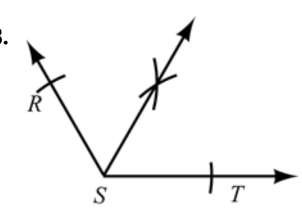
b. $m\angle 1 = 2(25) - 3 = 47^\circ$

16. **a.**
$$\begin{aligned} 2x - 3 + 6x - 1 &= 180 \\ 8x - 4 &= 180 \\ 8x &= 184 \\ x &= 23^\circ \end{aligned}$$

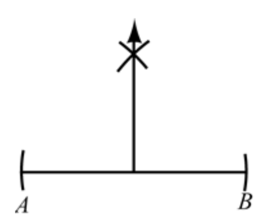
b. $m\angle 2 = 6(23) - 1 = 137^\circ$

17. $x + y = 90$

18.



19.



20. **1.** Given

2. Segment-Addition Postulate

3. Segment-Addition Postulate

4. Substitution

21. **1.** $2x - 3 = 17$

2. $2x = 20$

3. $x = 10$

22. **R1.** Given

S2. 90°

R3. Angle-Addition Postulate

S4. 90°

R5. Given

R6. Definition of Angle-Bisector

R7. Substitution

S8. $m\angle 1 = 45^\circ$

23. 108°