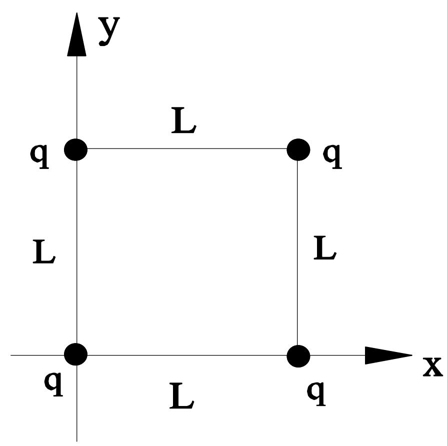
Chapter 1

Foundations of Electrostatics

Problem 1.1:



(a) The force on the upper right hand charge due to the other three charges at the corners of a square with sides of length L is

$$\mathbf{F} = \frac{q^2}{L^2}\hat{\mathbf{i}} + \frac{q^2}{L^2}\hat{\mathbf{j}} + \frac{q^2}{2L^2}\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)$$

$$= q^2\left[\frac{1+2\sqrt{2}}{2\sqrt{2}L^2}\right](\hat{\mathbf{i}}+\hat{\mathbf{j}}). \quad (1.1)$$

The magnitude of this force on any of the four charges is

$$F = q^2(1+2\sqrt{2})/2L^2. \quad (1.2)$$

(b) If the four charges are released from rest, we can write for the acceleration of any of the charges

$$\frac{F}{m} = a = \frac{dv}{dt} = \frac{dv}{dL'}\frac{dL'}{dt}, \tag{1.3}$$

where L' is the side length of the square at any time during the motion. The center of the square remains fixed, and the distance, r, of a charge from the center is related to L' by $L' = \sqrt{2}r$, so

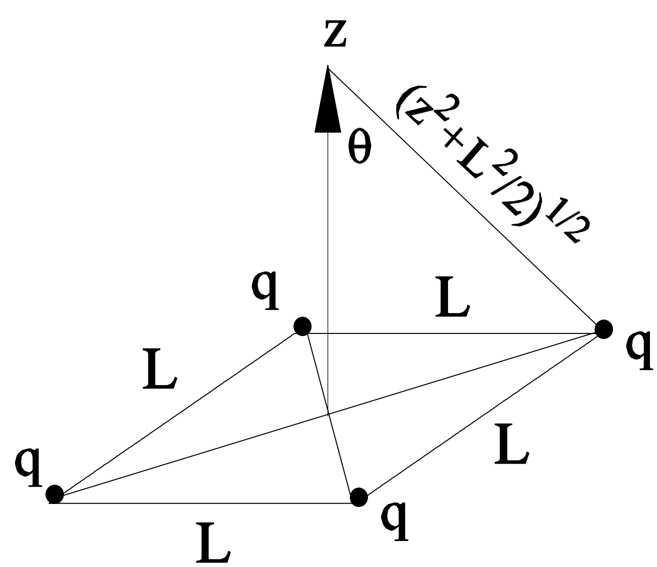
$$\frac{dL'}{dt} = \sqrt{2}\frac{dr}{dt} = \sqrt{2}v. \quad \text{Then, } a = \sqrt{2}v\frac{dv}{dL'}, \tag{1.4}$$

and the squared velocity after a long time is given by

$$V^2 = \int_L^\infty \sqrt{2}a\,dL' = \left[\frac{q^2(1+2\sqrt{2})}{\sqrt{2}m}\right]\int_L^\infty \frac{dL'}{L'^2} = \frac{q^2(1+2\sqrt{2})}{\sqrt{2}mL}. \tag{1.5}$$

The velocity is the square root of this.

Problem 1.2:



(a) The four point charges q are located at the corners of a square with sides of length L. The distance from each charge to a point z above the square, on the perpendicular axis of the square, is $\sqrt{z^2 + L^2/2}$. The horizontal fields cancel, and the magnitude of the vertical field is given by

$$E_z = E\cos\theta = \frac{4qz}{(z^2 + L^2/2)^{\frac{3}{2}}}. \tag{1.6}$$

(b) For small oscillations, $z << L$, and we can approximate the force on a charge $-q$ as

$$F = -\frac{8\sqrt{2}q^2 z}{L^3} = -kz. \tag{1.7}$$

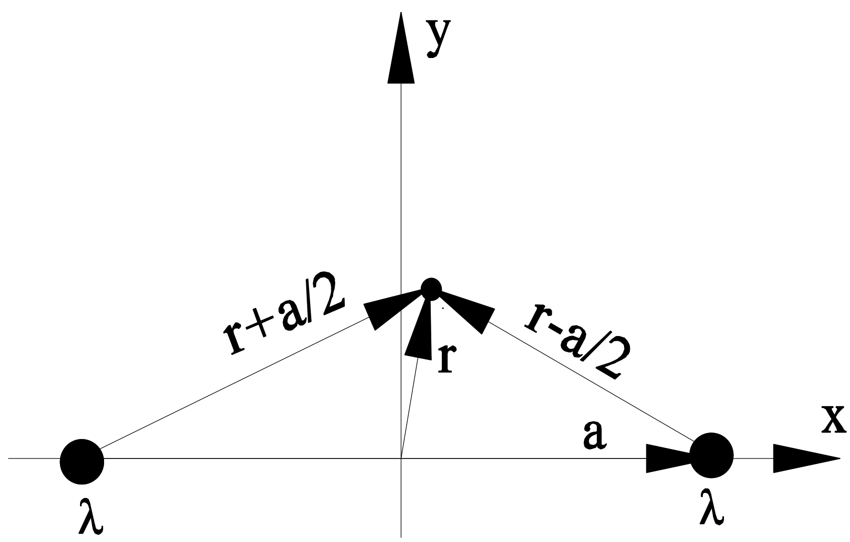
This is a restoring force proportional to the distance with an effective spring constant

$$k = \frac{8\sqrt{2}q^2}{L^3}, \tag{1.8}$$

which leads to simple harmonic motion with period

$$T = 2\pi\sqrt{\frac{m}{k}} - = 2\pi\left[\frac{mL^3}{8\sqrt{2}q^2}\right]^{\frac{1}{2}}. \tag{1.9}$$

Problem 1.3:



(a) By symmetry, the electric field of a long straight wire is perpendicular to the wire. Its magnitude a distance r from the wire is given by

$$E_z = \int_{-\infty}^{\infty} \frac{r\lambda dz}{(z^2+r^2)^{3/2}} = \frac{\lambda}{r}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{2\lambda}{r}. \tag{1.10}$$

We made the substitution $z = r\tan\theta$ in doing the integral. For the configuration of two wires a distance **a** apart, the electric field is

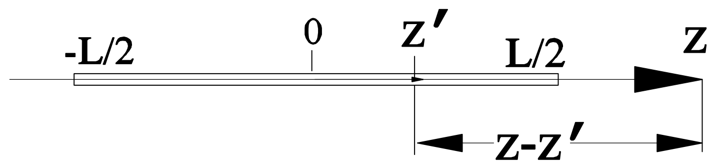
$$\mathbf{E} = \frac{2\lambda(\mathbf{r}-\mathbf{a}/2)}{|\mathbf{r}-\mathbf{a}/2)|^2} + \frac{2\lambda(\mathbf{r}+\mathbf{a}/2)}{|\mathbf{r}+\mathbf{a}/2)|^2}. \tag{1.11}$$

(b) The electric field in Cartesian coordinates is

$$E_x = \frac{2\lambda(x-a/2)}{(x-a/2)^2+y^2} + \frac{2\lambda(x+a/2)}{(x+a/2)^2+y^2} \tag{1.12}$$

$$E_y = \frac{2\lambda y}{(x-a/2)^2+y^2} + \frac{2\lambda y}{(x+a/2)^2+y^2}. \tag{1.13}$$

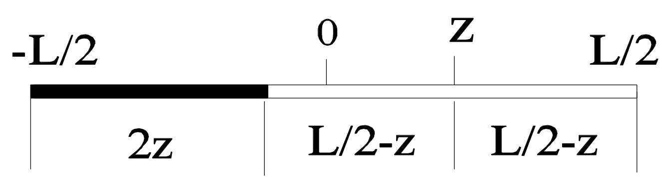
Problem 1.4:



(a) The field on the axis of the uniformly charged wire, a distance z from the center of the wire, is given for $z > L/2$ by

$$\begin{aligned} E_z &= \frac{Q}{L}\int_{-L/2}^{+L/2}\frac{dz'}{(z-z')^2} \\ &= \frac{Q}{L}\left[\frac{1}{z-L/2}-\frac{1}{z+L/2}\right] \\ &= \frac{Q}{z^2-L^2/4}. \end{aligned} \tag{1.14}$$

(b) For $-L/2 < z < L/2$, the point z is a distance $(L/2 - z)$ from the end of the wire. The wire can be thought of as two parts.



The part of the wire from $z' = z-(L/2-z) = 2z-L/2$ to $z' = 2L$ is symmetric about the point z. This means that the field due to that portion of the wire will cancel. The remaining part of the wire has a length $L' = L - 2(L/2 - z) = 2z$ and a charge $Q' = 2zQ/L$. The midpoint of this part of the wire is at $z_0 = (2z-L/2-L/2)/2 = z-L/2$. Thus the electric field from this part of the wire is

$$\begin{aligned} E_z &= \frac{Q'}{[(z-z_0)^2-L'^2/4]} \\ &= \frac{2Qz}{L[(z-z+L/2)^2-z^2]} \\ &= \frac{2Qz}{L[L^2/4-z^2]}. \end{aligned} \tag{1.15}$$