Chapter 1

1.1

(a) $\mathrm{m} = \dfrac{30\ \mathrm{lb}}{5.32\ \mathrm{ft/s^2}} = 5.639\ \mathrm{slugs}$ ⧫ $(133.5\ \mathrm{N})/(1.62\ \mathrm{m/s^2}) = 82.4\ \mathrm{kg}$

(b) $\mathrm{W} = \mathrm{mg} = (5.639)(32.2) = 181.6\ \mathrm{lb}$ ⧫ $(82.4\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) = 808\ \mathrm{N}$

1.2

$$W = \rho g V = (7850)(9.81)\left[\pi(0.04^2)(0.110)\right] = 42.58\ \mathrm{N} \blacktriangleleft$$

1.3

(a) $1\ \mathrm{kN}\cdot\mathrm{mm} = 1\ \mathrm{kN}\cdot\mathrm{mm} \times \dfrac{1000\ \mathrm{N}}{1.0\ \mathrm{kN}} \times \dfrac{1\ \mathrm{m}}{1000\ \mathrm{mm}} = 1\ \mathrm{N}\cdot\mathrm{m} \blacktriangleleft$

1.4

$$\begin{aligned} 30\ \mathrm{m/mL} &= \frac{30\ \mathrm{m}}{\mathrm{mL}} \times \frac{1\ \mathrm{km}}{1000\ \mathrm{m}} \frac{1000\ \mathrm{mL}}{1\ \mathrm{L}} \\ &= 30\ \mathrm{km/L} \blacktriangleleft \end{aligned}$$

1.5

$$\begin{aligned} E &= \frac{1}{2}(1000\ \mathrm{kg})\left(6\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = 18\ 000\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2} = 18\ 000\left(\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}\right)(\mathrm{m}) \\ &= 18\ 000\ \mathrm{N}\cdot\mathrm{m} = 18\ \mathrm{kN}\cdot\mathrm{m} \blacktriangleleft \end{aligned}$$

1.6

The dimensions of $\dfrac{gkx}{W}$ are: $[g][k][x]\left[\dfrac{1}{W}\right] = \left[\dfrac{L}{T^2}\right]\left[\dfrac{F}{L}\right][L]\left[\dfrac{1}{F}\right] = \left[\dfrac{L}{T^2}\right] = [a]$ Q.E.D.

1.7

The dimensions of $k = \dfrac{F}{x}$ are: $[k] = \left[\dfrac{F}{x}\right] = \left[\dfrac{ML}{T^2}\right]\left[\dfrac{1}{L}\right] = \left[\dfrac{M}{T^2}\right]$ ⧫

1.8

$$8\ \mathrm{mm}/\mu\mathrm{s} = \frac{8\ \mathrm{mm}}{\mu\mathrm{s}} \times \frac{1.0\ \mathrm{m}}{1000\ \mathrm{mm}} \times \frac{1.0\ \mu\mathrm{s}}{10^{-6}\mathrm{s}} = 8000\ \mathrm{m/s} \blacktriangleleft$$

1.9

$\mathrm{y} = \mathrm{kx}^2$ (where $\mathrm{k} = 1.0$)

The dimensions of $\mathrm{k} = \dfrac{\mathrm{y}}{\mathrm{x}^2}$ are : $\therefore [\mathrm{k}] = \left[\dfrac{\mathrm{y}}{\mathrm{x}^2}\right] = \left[\dfrac{\mathrm{L}}{\mathrm{L}^2}\right] = \left[\dfrac{1}{\mathrm{L}}\right]$

$\mathrm{y} = \mathrm{x}^2$ can be dimensionally correct if the units of the constant 1.0 (not shown explicity) are understood to be m^{-1}.

1.10

$$\left[\frac{L}{T^2}\right] = [A]\left[L^2\right] + [B]\,[L]\,[T]$$

$$\therefore [A] = \left[\frac{1}{LT^2}\right] \blacktriangleleft \qquad [B] = \left[\frac{1}{T^3}\right] \blacktriangleleft$$

1.11

(a) The dimensions of $x = At^2 - Bvt$ are

$$[L] = [A][T^2] - [B][LT^{-1}][T]$$
$$\therefore [A] = [LT^{-2}] \blacktriangleleft \quad [B] = [1](\text{dimensionless}) \blacktriangleleft$$

(b) The dimensions of $x = Avte^{-Bt}$ are

$$[L] = [A][LT^{-1}][T]e^{[e][T]}$$
$$[B][T] = [1] \quad \therefore [B] = [T^{-1}] \blacktriangleleft$$
$$[L] = [A][LT^{-1}][T] \quad \therefore [A] = [1] \blacktriangleleft$$

1.12

$$\left[\frac{d^4y}{dx^4}\right] = \left[\frac{L}{L^4}\right] = [L^{-3}]$$
$$\left[\frac{\omega^2\gamma}{D}y\right] = \frac{[T^{-2}][ML^{-1}]}{[FL^2]}[L] = \left[\frac{M}{T^2FL^2}\right]$$

Substituting $[F] = [MLT^{-2}]$ —see Eq. (1.2b)— we get

$$\left[\frac{\omega^2\gamma}{D}y\right] = \left[\frac{M}{T^2L^2}\right]\left[\frac{T^2}{ML}\right] = [L^{-3}] \text{ Q.E.D.}$$

Substituting $[F] = [MLT^{-2}]$ —see Eq. (1.2b)— we get

$$\left[\frac{\omega^2\gamma}{D}y\right] = \left[\frac{M}{T^2L^2}\right]\left[\frac{T^2}{ML}\right] = [L^{-3}] \text{ Q.E.D.}$$

1.13

The argument of the sine function must be dimensionless:

$$\left[\frac{Bx}{k}\right] = [1] \qquad [B][L]\left[\frac{L}{F}\right] = [1] \qquad [B] = [FL^{-2}] \blacktriangleleft$$
$$[F] = [Akx^2] = [A][FL^{-1}][L^2] \qquad [A] = [L^{-1}] \blacktriangleleft$$

1.14

$$110 \text{ kJ/s} = 110 \text{ kJ/s} \times \frac{1 \text{ kW}}{1 \text{ kJ/s}} = 110 \text{ kW} \blacktriangleleft$$

1.15

$$F = G\frac{m_A m_B}{R^2} = (6.67\times10^{-11})\frac{(12)(12)}{0.4^2} = 6.003\times10^{-8}\text{ N}$$
$$W = mg = (12)(9.81) = 117.7\text{ N}$$
$$\%\text{ of weight} = \frac{F}{W}\times100\% = \frac{6.003\times10^{-8}}{117.7}\times100\% = 5.10\times10^{-8}\,\% \quad \blacktriangleleft$$

1.16

$$\mathrm{F} = \mathrm{G}\frac{\mathrm{m_A m_B}}{\mathrm{R}^2} = \left(6.67\times10^{-11}\right)\frac{(8.9\text{ N}/9.81)(8.9\text{ N}/9.81)}{(0.406)^2} = 3.38\times10^{-10}\text{ N} \ \blacklozenge$$

1.17

$$\text{On earth:}\quad W_e = \frac{GM_e m}{R_e^2} \qquad \text{At elevation } h:\quad W = \frac{GM_e m}{(R_e+h)^2}$$
$$W = W_e\frac{R_e^2}{(R_e+h)^2} = 756\frac{6378^2}{(6378+8.5)^2} = 755\text{ N} \quad \blacktriangleleft$$

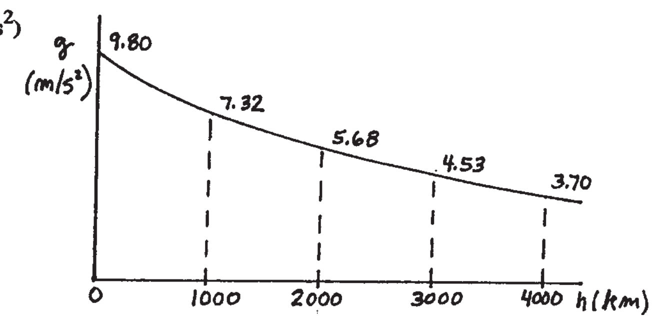
1.18

$$g_m = \frac{GM_m}{R_m^2} \qquad g_e = \frac{GM_e}{R_e^2}$$
$$\frac{g_m}{g_e} = \frac{M_m R_e^2}{M_e R_m^2} = \frac{0.07348(6378)^2}{5.974(1737)^2} = 0.1658 \approx \frac{1}{6}\text{ Q.E.D.}$$

1.19

Shown below is the plot of $g = \frac{GM_e}{R^2} = \frac{(6.67\times10^{-11})(5.9742\times10^{24})}{(6378+h)^2(10^6)}$

h (km)	g (m/s^2)
0	9.80
1000	7.32
2000	5.68
3000	4.53
4000	3.70



1.20

$$\text{On earth:}\quad W_e = \frac{GM_e m}{R_e^2} \quad \text{At elevation } h:\quad W = \frac{GM_e m}{(R_e+h)^2}$$

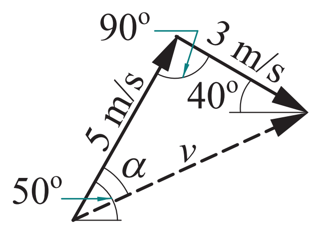
$$W = \frac{W_e}{10} \quad \frac{GM_e m}{(R_e+h)^2} = \frac{GM_e m}{10R_e^2} \quad (R_e+h)^2 = 10R_e^2$$
$$(6378+h)^2 = 10(6378)^2 \quad h = 13\ 790\text{ km} \quad \blacktriangleleft$$

1.21

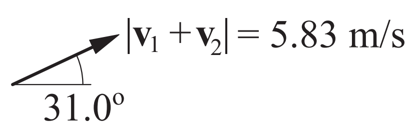
$$R = R_e + R_m + d = 6378 + 1737 + 387 \times 10^3$$
$$= 392.1 \times 10^3 \text{ km} = 392.1 \times 10^6 \text{ m}$$

$$F = G\frac{M_e M_m}{R^2} = \left(6.67 \times 10^{-11}\right) \frac{(5.974 \times 10^{24})(0.07348 \times 10^{24})}{(392.1 \times 10^6)^2}$$
$$= 1.904 \times 10^{20} \text{ N} ◀$$

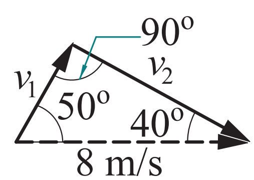
1.22



$$v = \sqrt{5^2 + 3^2} = 5.83 \text{ m/s} \quad \alpha = \tan^{-1}\frac{3}{5} = 31.0^\circ$$

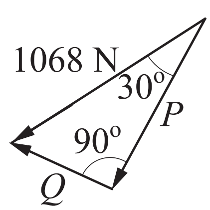


1.23



$$v_1 = 8\sin 40^\circ = 5.14 \text{ m/s} ◀ \qquad v_2 = 8\sin 50^\circ = 6.13 \text{ m/s} ◀$$

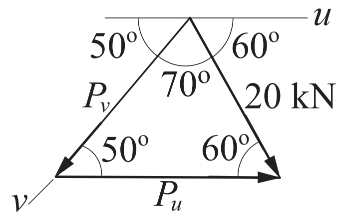
1.24



Component parallel to AB : $P = 1068\cos 30^\circ = 924.9$ N ◀

Component Pependicular to AB : $Q = 1068\sin 30^\circ = 534.0$ N ◀

1.25

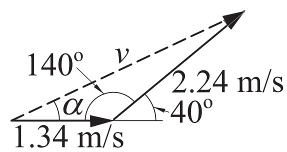


$$\frac{P_v}{\sin 60^\circ} = \frac{P_u}{\sin 70^\circ} = \frac{20}{\sin 50^\circ}$$

$$P_v = 20\frac{\sin 60^\circ}{\sin 50^\circ} = 22.6 \text{ kN} \blacktriangleleft$$

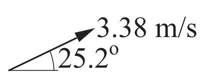
$$P_u = 20\frac{\sin 70^\circ}{\sin 50^\circ} = 24.5 \text{ kN} \blacktriangleleft$$

1.26

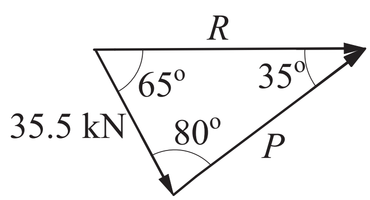


Law of cosines: $v = \sqrt{1.34^2 + 2.24^2 - 2(1.34)(2.24)\cos 140^\circ} = 3.378 \text{ m/s}$

Law of sines: $\dfrac{2.24}{\sin\alpha} = \dfrac{3.378}{\sin 140^\circ} \quad \sin\alpha = 0.4262 \quad \alpha = 25.2^\circ$

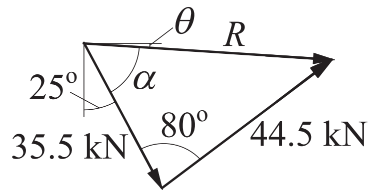


1.27



$$\frac{P}{\sin 65^\circ} = \frac{35.5}{\sin 35^\circ} \qquad P = 35.5\frac{\sin 65^\circ}{\sin 35^\circ} = 56.1 \text{ kN} \blacktriangleleft$$

1.28

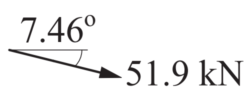


Law of cosines:

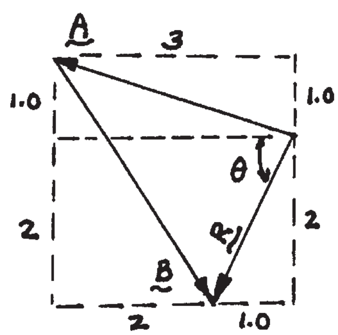
$$R = \sqrt{35.5\ \text{kN}^2 + 44.5\ \text{kN}^2 - 2(35.5\ \text{kN})(44.5\ \text{kN})\cos 80^\circ}$$
$$= 51.9\ \text{kN} \quad \blacktriangleleft$$

Law of sines:

$$\frac{44.5}{\sin\alpha} = \frac{51.9}{\sin 80^\circ} \qquad \sin\alpha = \frac{44.5}{51.9}\sin 80^\circ = 0.8444$$
$$\alpha = \sin^{-1}(0.8438) = 57.61^\circ$$
$$\theta = 90^\circ - 25^\circ - 57.61^\circ = 7.39^\circ \quad \blacktriangleleft$$



1.29

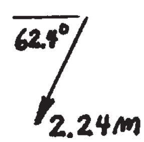


R = A + B

By inspection of the triangle (dimensions in meters

$R = \sqrt{2^2 + 1.0^2} = 2.24$ m and $\theta = \tan^{-1} 2 = 62.4$'

Therefore, the resultant of A and B is:

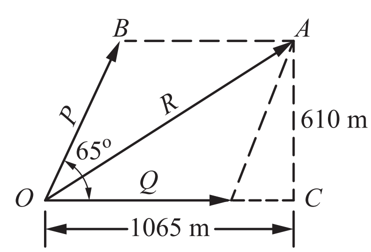


♦

1.30

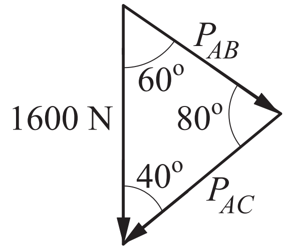
R = P + Q

$$P = \frac{610}{\sin 65^\circ} = 673\ \text{m}$$
$$Q = 1065 - P\cos 65^\circ$$
$$= 1065 - 673\cos 65^\circ = 781\ \text{m}$$



Therefore, the components are: 673 m along OB and 781 m along OC ♦

1.31



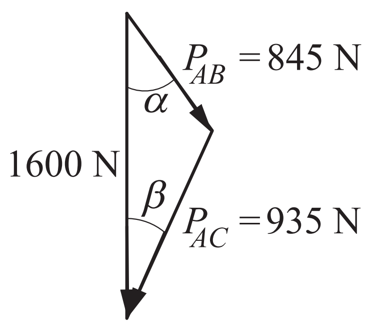
Law of sines:

$$\frac{1600}{\sin 80^\circ} = \frac{P_{AB}}{\sin 40^\circ} = \frac{P_{AC}}{\sin 60^\circ}$$

$$P_{AB} = \frac{1600 \sin 40^\circ}{\sin 80^\circ} = 1044 \text{ N} \quad ◀$$

$$P_{AC} = \frac{1600 \sin 60^\circ}{\sin 80^\circ} = 1407 \text{ N} \quad ◀$$

1.32



Law of cosines:

$$935^2 = 1600^2 + 845^2 - 2(1600)(845)\cos\alpha$$

$$\alpha = 27.4^\circ \quad ◀$$

$$845^2 = 1600^2 + 935^2 - 2(1600)(935)\cos\beta$$

$$\beta = 24.6^\circ \quad ◀$$

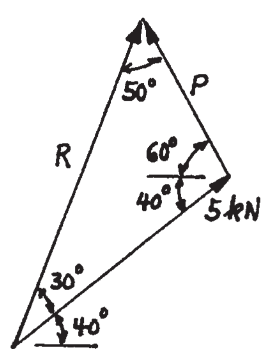
1.33

Law of sines:

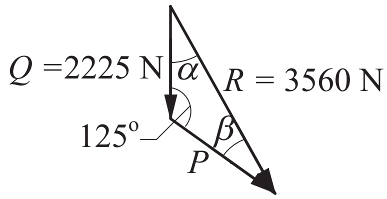
$$\frac{P}{\sin 30^\circ} = \frac{5}{\sin 50^\circ}$$

which gives

$$P = \frac{5 \sin 30^\circ}{\sin 50^\circ} = 3.26 \text{ kN} \quad ◆$$

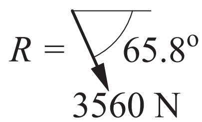


1.34



Law of sines:

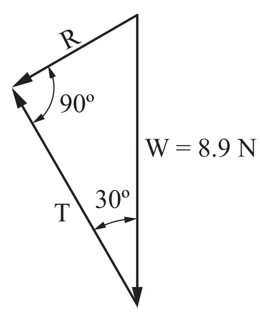
$$\frac{2225}{\sin\beta} = \frac{3560}{\sin 125^\circ} \qquad \beta = 30.8^\circ$$
$$\alpha = 180^\circ - (125^\circ + 30.8^\circ) = 24.2^\circ$$



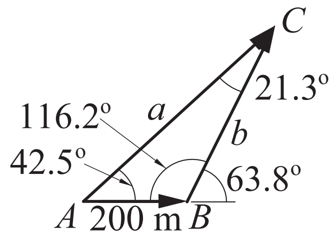
$$\frac{3560}{\sin 125^\circ} = \frac{P}{\sin 24.2^\circ} \qquad P = 1780 \text{ N} \blacktriangleleft$$

1.35

$$\text{R} = \text{W} + \text{T}$$
$$\text{T} = 8.9\cos 30^\circ = 7.708 \text{ N} \blacklozenge$$



1.36

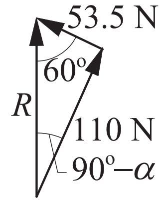


$$\text{Law of sines:} \quad \frac{200}{\sin 21.3^\circ} = \frac{a}{\sin 116.2^\circ} = \frac{b}{\sin 42.5^\circ}$$

$$\therefore a = \frac{200 \sin 116.2^\circ}{\sin 21.3^\circ} = 494 \text{ m} \blacktriangleleft$$

$$b = \frac{200 \sin 42.5^\circ}{\sin 21.3^\circ} = 372 \text{ m} \blacktriangleleft$$

1.37

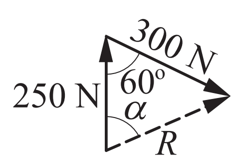


$$\frac{53.5}{\sin(90^\circ - \alpha)} = \frac{110}{\sin 60^\circ}$$

$$\sin(90^\circ - \alpha) = \frac{53.5 \sin 60^\circ}{110} = 0.4212$$

$$90^\circ - \alpha = 24.91^\circ \qquad \alpha = 65.1^\circ \blacktriangleleft$$

***1.38**



First compute the resultant **R** of the two known forces. The smallest required **F** has the same direction as **R** and its magnitude is 500 N − R.

$$\text{Law of consines:} \quad R = \sqrt{250^2 + 300^2 - 2(250)(300)\cos 60^\circ}$$

$$= 278.4 \text{ N}$$

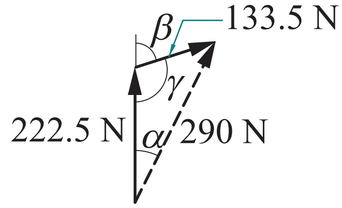
$$\therefore F = 500 - 278.4 = 222 \text{ N}$$

$$\text{Law of sines:} \quad \frac{300}{\sin \alpha} = \frac{278.4}{\sin 60^\circ}$$

$$\alpha = \sin^{-1} \frac{300 \sin 60^\circ}{278.4} = 68.9^\circ$$

◀

1.39



Law of cosines: $290^2 = 222.5^2 + 133.5^2 - 2(222.5)(133.5)\cos\gamma$

$$\gamma = \cos^{-1}\frac{290^2 + 222.5^2 + 133.5^2}{2(222.5)(133.5)} = 106.38°$$

$$\beta = 180° - \gamma = 180° - 106.38° = 73.6° \blacktriangleleft$$

Law of sines: $\dfrac{133.5}{\sin\alpha} = \dfrac{290}{\sin 106.38°}$

$$\alpha = \sin^{-1}\frac{133.5\sin 106.38°}{290} = 26.2° \blacktriangleleft$$

1.40

$$P = -135\cos 50°\sin 30°\mathrm{i} + 135\cos 50°\cos 30°\mathrm{j} + 135\sin 50°\ \mathrm{k}\ \mathrm{N}$$

$$\therefore P = -43.4\ \mathrm{i} + 75.2\ \mathrm{j} + 130.4\ \mathrm{k}\ \mathrm{N}\blacklozenge$$

1.41

(a) $\mathbf{r} = 240\sin 40^\circ\cos 50^\circ\,\mathbf{i} + 240\sin 40^\circ\sin 50^\circ\,\mathbf{j} + 240\cos 40^\circ\,\mathbf{k}$ mm

$\therefore\ \mathbf{r} = 99.16\mathbf{i} + 118.2\mathbf{j} + 183.8\mathbf{k}$ mm ♦

(b) $\lambda_x = \dfrac{r_x}{r} = \dfrac{99.16}{240} = 0.413 \qquad \lambda_y = \dfrac{r_y}{r} = \dfrac{118.2}{240} = 0.493 \qquad \lambda_z = \dfrac{r_z}{r} = \dfrac{183.8}{240} = 0.766$

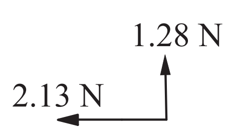
$\therefore\ \vec{\lambda} = 0.413\mathbf{i} + 0.493\mathbf{j} + 0.766\mathbf{k}$ ♦

1.42

$\overrightarrow{AB} = -1.52\mathbf{i} + 0.915\mathbf{j}$ m $\qquad \left|\overrightarrow{AB}\right| = \sqrt{1.52^2 + 0.915^2} = 1.774$ m

$$\lambda = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{-1.52\mathbf{i} + 0.915\mathbf{j}}{1.774} = -0.8568\mathrm{i} + 0.5159\mathbf{j}$$

$$\mathbf{F} = F\boldsymbol{\lambda} = 2.49(-0.8568\mathbf{i} + 0.5159\mathbf{j}) = -2.13\mathbf{i} + 1.28\mathbf{j}\ \mathrm{N} \blacktriangleleft$$



1.43

(a) $\overrightarrow{\mathrm{AB}} = 7\mathrm{i} + \mathrm{j} + 5\ \mathrm{k}\ \mathrm{m} \qquad \therefore |\overrightarrow{\mathrm{AB}}| = \sqrt{7^2 + 1^2 + 5^2} = 8.66\ \mathrm{m}$ ⧫

(b) $\vec{\lambda}_{\mathrm{AB}} = \dfrac{\overrightarrow{\mathrm{AB}}}{|\overrightarrow{\mathrm{AB}}|} = \dfrac{7\mathbf{i} + \mathbf{j} + 5\mathbf{k}}{8.66} = 0.808\mathbf{i} + 0.115\mathbf{j} + 0.577\mathbf{k}\ \mathrm{m}$ ⧫

1.44

(a) $\boldsymbol{\lambda}_{AB} = \dfrac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \dfrac{-2.2\mathbf{i} + 7.5\mathbf{j} + 3\mathbf{k}}{8.372} = -0.2628\mathbf{i} + 0.8958\mathbf{j} + 0.3583\mathbf{k}$ ◀

(b) $\mathbf{v} = 8\boldsymbol{\lambda}_{AB} = 8(-0.2628\mathbf{i} + 0.8958\mathbf{j} + 0.3583\mathbf{k})$
$= -2.10\mathbf{i} + 7.17\mathbf{j} + 2.87\mathbf{k}\ \mathbf{m/s}$ ◀

1.45

$$\boldsymbol{\lambda}_{OA} = \frac{\overrightarrow{OA}}{\left|\overrightarrow{OA}\right|} = \frac{-3\mathbf{i} + 4\mathbf{j} + 2.5\mathbf{k}}{5.590} = -0.5367\mathbf{i} + 0.7156\mathbf{j} + 0.4472\mathbf{k}$$

$$\mathbf{F} = F\lambda_{OA} = 320(-0.5367\mathbf{i} + 0.7156\mathbf{j} + 0.4472\mathbf{k})$$
$$= -172\mathbf{i} + 229\mathbf{j} + 143\mathbf{k}\ \mathrm{N} \blacktriangleleft$$

1.46

$$\boldsymbol{\lambda}_{BA} = \frac{\overrightarrow{BA}}{\left|\overrightarrow{BA}\right|} = \frac{4.27\mathbf{i} - 3.05\mathbf{j} - 5.5\mathbf{k}}{7.602} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

$$\mathbf{F} = F\boldsymbol{\lambda}_{AB} = 712(0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k})$$
$$= 399.9\mathbf{i} - 285.7\mathbf{j} - 515.1\mathbf{k}\ \mathrm{N} \blacktriangleleft$$

1.47

$$\overrightarrow{AB} = 48.8\mathbf{i} + 67\mathbf{j} - 21.3\mathbf{k}\ \mathrm{m}$$

$$\boldsymbol{\lambda} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{48.8\mathbf{i} + 67\mathbf{j} - 21.3\mathbf{k}}{\sqrt{48.8^2 + 67^2 + 21.3^2}} = 0.5696\mathbf{i} + 0.7832\mathbf{j} - 0.2492\mathbf{k}$$

$$\mathbf{v} = v\boldsymbol{\lambda} = 427(0.5696\mathbf{i} + 0.7832\mathbf{j} - 0.2492\mathbf{k})$$
$$= 106.3\mathbf{i} + 334.3\mathbf{j} - 106.3\mathbf{k}\ \mathrm{m/s} \blacktriangleleft$$

1.48

(a)

$$\overrightarrow{BA} = -6.1\mathbf{i} + 18.3\mathbf{j} - 27.4\mathbf{k}\ \mathrm{ft} \qquad \left|\overrightarrow{BA}\right| = \sqrt{20^2 + 60^2 + 90^2} = 33.5\ \mathrm{m}$$

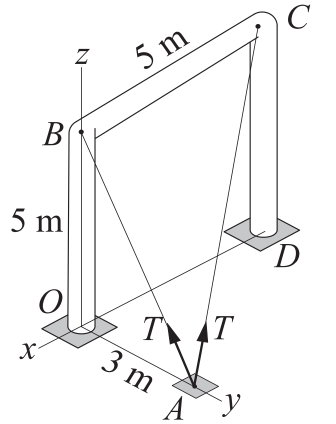
$$\boldsymbol{\lambda} = \frac{\overrightarrow{BA}}{\left|\overrightarrow{BA}\right|} = \frac{-6.1\mathbf{i} + 18.3\mathbf{j} - 27.4\mathbf{k}}{33.5} = -0.1818\mathbf{i} + 0.5455\mathbf{j} - 0.8182\mathbf{k}$$

$$F_x = F\lambda_x = 2.67\,(-0.1818) = -486 \text{ N} \blacktriangleleft$$
$$F_y = F\lambda_y = 2.67(0.5455) = 1458 \text{ N} \blacktriangleleft$$
$$F_z = F\lambda_z = 2.67(-0.8182) = -2184 \text{ N} \blacktriangleleft$$

(b)

$$\theta_x = \cos^{-1}\lambda_x = \cos^{-1}(-0.1818) = 100.5^\circ \quad \blacktriangleleft$$
$$\theta_y = \cos^{-1}\lambda_y = \cos^{-1}(0.5455) = 56.9^\circ \quad \blacktriangleleft$$
$$\theta_z = \cos^{-1}\lambda_z = \cos^{-1}(-0.8182) = 144.9^\circ \quad \blacktriangleleft$$

1.49



$$\overrightarrow{AB} = -3\mathbf{j} + 5\mathbf{k} \text{ m} \qquad \left|\overrightarrow{AB}\right| = \sqrt{3^2 + 5^2} = 5.831 \text{ m}$$
$$\mathbf{T}_{AB} = T\frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = 35\frac{-3\mathbf{j} + 5\mathbf{k}}{5.831} = -18.01\mathbf{j} + 30.01\mathbf{k} \text{ kN}$$
$$\overrightarrow{AC} = -5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} \text{ m} \qquad \left|\overrightarrow{AC}\right| = \sqrt{5^2 + 3^2 + 5^2} = 7.681 \text{ m}$$
$$\mathbf{T}_{AC} = T\frac{\overrightarrow{AC}}{\left|\overrightarrow{AC}\right|} = 35\frac{-5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}}{7.681} = -22.78\mathbf{i} - 13.67\mathbf{j} + 22.78\mathbf{k} \text{ kN}$$

$$\begin{aligned}\mathbf{R} &= \mathbf{T}_{AB} + \mathbf{T}_{AC} \\ &= -22.78\mathbf{i} + (-18.01 - 13.67)\mathbf{j} + (30.01 + 22.78)\mathbf{k} \\ &= -22.8\mathbf{i} - 31.7\mathbf{j} + 52.8\mathbf{k} \text{ kN} \blacktriangleleft\end{aligned}$$

1.50

$$\overrightarrow{AB} = 2.44\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k} \text{ m} \qquad \left|\overrightarrow{AB}\right| = \sqrt{2.44^2 + 2.44^2 + 1.22^2} = 3.66 \text{ m}$$

$$\mathbf{F}_{AB} = F\frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = F\frac{2.44\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k}}{3.66}$$

$$\overrightarrow{AC} = -1.22\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k} \text{ ft} \qquad \left|\overrightarrow{AC}\right| = \sqrt{1.22^2 + 2.44^2 + 1.22^2} = 2.99 \text{ m}$$

$$\mathbf{F}_{AC} = 890\frac{\overrightarrow{AC}}{\left|\overrightarrow{AC}\right|} = 890\frac{-1.22\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k}}{2.99}$$

The resultant $\mathbf{R}$ lies in the yz-plane if

$$R_x = (F_{AB})_x + (F_{AC})_x = 0 \qquad F\frac{2.44}{3.66} - 890\frac{1.22}{2.99} = 0$$

$$F = 545 \text{ N} \blacktriangleleft$$

1.51

(a)

$$\begin{aligned}\mathbf{R} &= (F_1 + F_2 \sin 35°)\mathbf{i} + (F_2 \cos 35° + F_3 \cos 65°)\mathbf{j} + (F_3 \sin 65°)\mathbf{k} \\ &= (1.6 + 1.2 \sin 35°)\mathbf{i} + (1.2 \cos 35° + 1.0 \cos 65°)\mathbf{j} + (1.0 \sin 65°)\mathbf{k} \\ &= 2.288\mathbf{i} + 1.4056\mathbf{j} + 0.9063\mathbf{k} \text{ kN} \blacktriangleleft\end{aligned}$$

(b)

$$R = \sqrt{2.288^2 + 1.4056^2 + 0.9063^2} = 2.834 \text{ kN}$$

$$\boldsymbol{\lambda} = \frac{\mathbf{R}}{R} = \frac{2.288\mathbf{i} + 1.4056\mathbf{j} + 0.9063\mathbf{k}}{2.834}$$

$$= 0.807\mathbf{i} + 0.496\mathbf{j} + 0.320\mathbf{k}$$

$$\therefore R = 2.83(0.807\mathbf{i} + 0.496\mathbf{j} + 0.320\mathbf{k}) \text{ kN} \blacktriangleleft$$

1.52

$$\text{P} = 534\left(\frac{4\text{i} + 3\text{j}}{5}\right) = 427\text{i} + 320\text{j N} \qquad \text{Q} = 578\left(\frac{5\text{i} - 12\text{j}}{13}\right) = 222\text{i} - 534\text{j N}$$

$$\therefore \text{P} + \text{Q} = (427\text{i} + 320\text{j}) + (222\text{i} - 534\text{j}) = 649\text{i} - 214\text{j N} \blacklozenge$$

1.53

$$\mathbf{P} = 400\frac{4\mathbf{i} + 3\mathbf{j}}{5} = 320\mathbf{i} + 240\mathbf{j} \text{ N} \qquad \mathbf{NQ} = Q\frac{5\mathbf{i} - 12\mathbf{j}}{13}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$ lies in x-direction, we have

$$R_y = 0 \qquad 240 - \frac{12}{13}Q = 0 \qquad Q = 260 \text{ N} \blacktriangleleft$$

$$R = R_x = 320 + 260\frac{5}{13} = 420 \text{ N} \blacktriangleleft$$

1.54

$$\begin{aligned} P_x + Q_x &= R_x \qquad P\cos 30^\circ - Q\sin 30^\circ = 1600\cos 25^\circ \\ P_y + Q_y &= R_y \qquad P\sin 30^\circ - Q\cos 30^\circ = -1600\sin 25^\circ \end{aligned}$$

Solutioin is : $P = 3190$ N ◄ $\qquad Q = 2620$ N ◄

1.55

$$P_x + Q_x = R_x \quad 3\cos\theta = 2\sin 55^\circ$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\sin 55^\circ\right) = 56.90^\circ \quad ◄$$

$$P_y + Q_y = R_y \quad 3\sin\theta - Q = -2\sin 55^\circ$$

$$Q = 2\sin 55^\circ + 3\sin 56.90^\circ = 4.15 \text{ kN} \quad ◄$$

1.56

$$\boldsymbol{\lambda}_P = \frac{1.83\mathbf{i} + 2.44\mathbf{j} - 3.66\mathbf{k}}{\sqrt{1.83^2 + 2.44^2 + (-3.66)^2}} = 0.3841\mathbf{i} + 0.5121\mathbf{j} - 0.7682\mathbf{k}$$

$$\boldsymbol{\lambda}_Q = \frac{-1.83\mathbf{i} + 1.83\mathbf{j} - 3.66\mathbf{k}}{\sqrt{(-1.83^2) + 1.83^2 + (-3.66)^2}} = -0.4082\mathbf{i} + 0.4082\mathbf{j} - 0.8165\mathbf{k}$$

$$\boldsymbol{\lambda}_F = \frac{-2.44\mathbf{j} - 3.66\mathbf{k}}{\sqrt{(-2.44)^2 + (-3.66)^2}} = -0.5547\mathbf{j} - 0.8321\mathbf{k}$$

$$\begin{aligned} P_x + Q_x + F_x &= 0 \qquad 0.3841P - 0.4082Q + 0 = 0 \\ P_y + Q_y + F_y &= 0 \qquad 0.5121P + 0.4082Q - 0.5547(534\text{N}) = 0 \end{aligned}$$

Solution is: $P = 330$ N ◄ $\qquad Q = 310$ N ◄

1.57

(a) $\mathbf{A}\cdot\mathbf{B} = 12(-2) + 8(3) = 0$ ◄

(b) $\mathbf{A}\cdot\mathbf{B} = 5(7) = 35 \text{ N}\cdot\text{m}$ ◄

(c) $\mathbf{A}\cdot\mathbf{B} = 3(-6) + 2(2) + (-1)(-8) = -6 \text{ m}^2$ ◄

1.58

(a) $\mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 12 & 8 \\ 4 & -2 & 3 \end{vmatrix} = 52\mathbf{i} + 32\mathbf{j} - 48\mathbf{k} \text{ m}^2$ ◄

(b) $\mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ 7 & 0 & -12 \end{vmatrix} = -36\mathbf{i} + 60\mathbf{j} - 21\mathbf{k} \text{ N}\cdot\text{m}$ ◄

(c) $\mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ -6 & 2 & -8 \end{vmatrix} = -14\mathbf{i} + 30\mathbf{j} + 18\mathbf{k} \text{ m}^2$ ◄

1.59

$$\mathbf{r}\times\mathbf{F}\cdot\boldsymbol{\lambda}=\begin{vmatrix}4 & -6 & 2\\ 20 & 40 & -30\\ 0 & 0.8 & 0.6\end{vmatrix}=296\ \text{N}\cdot\text{m} \blacktriangleleft$$

$$\boldsymbol{\lambda}\times\mathbf{r}\cdot\mathbf{F}=\begin{vmatrix}0 & 0.8 & 0.6\\ 4 & -6 & 2\\ 20 & 40 & -30\end{vmatrix}=296\ \text{N}\cdot\text{m} \blacktriangleleft$$

1.60

$$\mathbf{A}=2\mathbf{i}+1.2\mathbf{j}\ \text{m}\quad \mathbf{B}=2\mathbf{i}+1.2\mathbf{j}+1.5\mathbf{k}\ \text{m}\quad \mathbf{C}=-1.5\mathbf{k}\ \text{m}$$

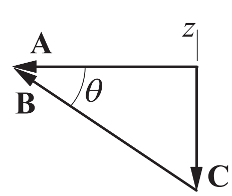
$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 1.2 & 0\\ 2 & 1.2 & 1.5\end{vmatrix}=1.8\mathbf{i}-3\mathbf{j}\ \text{m}^2 \blacktriangleleft$$

$$\mathbf{C}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & -1.5\\ 2 & 1.2 & 1.5\end{vmatrix}=1.8\mathbf{i}-3\mathbf{j}\ \text{m}^2 \blacktriangleleft$$

1.61

$$\mathbf{A}=2\mathbf{i}+1.2\mathbf{j}\ \text{m}\quad \mathbf{B}=2\mathbf{i}+1.2\mathbf{j}+1.5\mathbf{k}\ \text{m}$$
$$A=\sqrt{2^2+1.2^2}=2.332\ \text{m}\quad B=\sqrt{2^2+1.2^2+1.5^2}=2.773\ \text{m}$$

$$\cos\theta=\frac{\mathbf{A}\cdot\mathbf{B}}{AB}=\frac{2(2)+1.2(1.2)}{(2.332)(2.773)}=0.8412$$
$$\therefore\ \theta=32.7^\circ \blacktriangleleft$$



Because the three vectors form a right triangle, we have in this case

$$\theta=\cos^{-1}\frac{A}{B}=\cos^{-1}\frac{2.332}{2.773}=32.8^\circ$$

The difference in the results is due to round-off error.

1.62

$$B_z = A_z = \sqrt{4.27^2 + 2.74^2}\,\csc 50^\circ = 6.62 \text{ m}$$
$$\mathbf{A} = 2.74\mathbf{i} + 4.27\mathbf{j} + 6.62\mathbf{k} \text{ m} \qquad \mathbf{B} = 1.83\mathbf{i} + 6.62\mathbf{k} \text{ m}$$

$$A = \sqrt{2.74^2 + 4.27^2 + 6.62^2} = 8.34 \text{ m}$$
$$B = \sqrt{1.83^2 + 6.62^2} = 6.87 \text{ m}$$
$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{2.74(1.83) + 6.62(6.62)}{8.34(6.87)} = 0.8524$$
$$\theta = 31.5^\circ \quad \blacktriangleleft$$

1.63

Statement (ii) is true. ◆

<u>Proof</u> A × B is perpendicular to A and B, that is, normal to plane S. Therefore, C = A × (A × B) is "normal to the normal" of S. Then C lies in plane S. (Note: C is also normal to A.)

1.64

$$\mathbf{P} = 76.2\mathbf{i} + 101.6\mathbf{k} \text{ mm} \qquad \mathbf{Q} = -50.8\mathbf{j} + 101.6\mathbf{k} \text{ mm}$$
$$P = \sqrt{76.2^2 + 101.6^2} = 127 \text{ mm} \qquad Q = \sqrt{50.8^2 + 101.6^2} = 113.6 \text{ mm}$$

(a)

$$\cos\theta = \frac{\mathbf{P}\cdot\mathbf{Q}}{PQ} = \frac{76.2(-50.8) + 101.6(101.6)}{127(113.6)} = 0.4472$$
$$\theta = 63.4^\circ \quad \blacktriangleleft$$

(b)

$$\mathbf{P}\times\mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 76.2 & 0 & 101.6 \\ 0 & -50.8 & 101.6 \end{vmatrix} = 8\mathbf{i} - 12\mathbf{j} - 6\mathbf{k} \text{ mm}$$
$$\boldsymbol{\lambda} = \frac{\mathbf{P}\times\mathbf{Q}}{|\mathbf{P}\times\mathbf{Q}|} = \frac{8\mathbf{i} - 12\mathbf{j} - 6\mathbf{k}}{\sqrt{8^2 + 12^2 + 6^2}}$$
$$= 0.512\mathbf{i} - 0.768\mathbf{j} - 0.384\mathbf{k} \quad \blacktriangleleft$$

1.65

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 4 & -3 & -2\\ -2 & -4 & 3\end{vmatrix}=-17\mathbf{i}-8\mathbf{j}-22\mathbf{k}\ \text{m}^2$$

$$\boldsymbol{\lambda}=\frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}=\pm\frac{-17\mathbf{i}-8\mathbf{j}-22\mathbf{k}}{\sqrt{17^2+8^2+22^2}}$$
$$=\pm\left(-0.588\mathbf{i}-0.277\mathbf{j}-0.760\mathbf{k}\right)\ ◀$$

1.66

$$\overrightarrow{CA}=(0-3)\mathbf{i}+(-2-0)\mathbf{j}+(2-0)\mathbf{k}=-3\mathbf{i}-2\mathbf{j}+2\mathbf{k}\ \text{mm}$$
$$\overrightarrow{CB}=(-1-3)\mathbf{i}+(4-0)\mathbf{j}+(1-0)\mathbf{k}=-4\mathbf{i}+4\mathbf{j}+\mathbf{k}\ \text{mm}$$

$$\left|\overrightarrow{CA}\right|=\sqrt{(-3)^2+(-2)^2+2^2}=4.123\ \text{mm}$$
$$\left|\overrightarrow{CB}\right|=\sqrt{(-4)^2+4^2+1^2}=5.745\ \text{mm}$$
$$\overrightarrow{CA}\times\overrightarrow{CB}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -3 & -2 & 2\\ -4 & 4 & 1\end{vmatrix}=-10\mathbf{i}-5\mathbf{j}-20\mathbf{k}\ \text{mm}^2$$
$$\boldsymbol{\lambda}=\pm\frac{\overrightarrow{CA}\times\overrightarrow{CB}}{\left|\overrightarrow{CA}\times\overrightarrow{CB}\right|}=\pm\frac{-10\mathbf{i}-5\mathbf{j}-20\mathbf{k}}{4.123(5.745)}=\pm(-0.422\mathbf{i}-0.211\mathbf{j}-0.844\mathbf{k})\ ◀$$

1.67

$$\mathbf{P}=3\mathbf{i}+4\mathbf{k}\ \text{m}\qquad\mathbf{Q}=3\mathbf{i}+4\mathbf{j}+5\mathbf{k}\ \text{m}$$
$$\boldsymbol{\lambda}=\frac{\overrightarrow{OA}}{\left|\overrightarrow{OA}\right|}=\frac{3\mathbf{i}+4\mathbf{j}}{\sqrt{3^2+4^2}}=0.6\mathbf{i}+0.8\mathbf{j}$$

The component of $\mathbf{P}\times\mathbf{Q}$ in direction of $\boldsymbol{\lambda}$ is

$$\mathbf{P}\times\mathbf{Q}\cdot\boldsymbol{\lambda}=\begin{vmatrix}3 & 0 & 4\\ 3 & 4 & 5\\ 0.6 & 0.8 & 0\end{vmatrix}=-12.0\ \text{m}\ ◀$$

1.68

$$\vec{\lambda}_{\text{A}}=\frac{\text{A}}{\text{A}}=\frac{0.61\mathbf{i}+0.914\mathbf{j}-1.52\mathbf{k}}{\sqrt{1.876}}=0.3244\mathbf{i}-0.4867\mathbf{j}+0.8111\mathbf{k}$$
$$\text{F}_{\text{A}}=\text{F}\bullet\vec{\lambda}_{\text{A}}=26.7(0.3244)+89(-0.4867)+(-53.4)(0.8111)=-77.92\ \text{N}♦$$

1.69

$$\mathbf{A} \cdot \mathbf{B} = 0$$
$$3(4) - a(1) - 2(1) = 0 \qquad a = 10.0 \blacktriangleleft$$

***1.70**

$$\vec{\lambda}_B = \frac{B}{B} = \frac{152.4\mathbf{i} + 50.8\mathbf{k}}{\sqrt{152.4^2 + 50.8^2}} = 0.950\mathbf{i} + 0.313\mathbf{k}$$

The component of A parallel to B is : $A_B = A \bullet \vec{\lambda}_B = 3(76.2(0.950) - 102(0.313)) =$ 40.46 mm

$$\therefore A_B = 40.46(0.950\mathbf{i} - 0.313\mathbf{k}) \text{ mm} \blacklozenge$$

Letting A_C be the component of A that is perpendicular to B, we have

$A_C = A - A_B$
$A_C = (76.2\mathbf{i} + 127\mathbf{j} - 102\mathbf{k}) - 40.46(0.950\mathbf{i} - 0.313\mathbf{k}) = 37.8\mathbf{i} + 127\mathbf{j} - 114.7\mathbf{k}$ mm
$\therefore A_C = \sqrt{37.8^2 + 127^2 - 114.7^2} = 175.3$ mm

The Unit vector in the direction of A_C is

$$\vec{\lambda}_C = \frac{A_C}{A_C} = \frac{37.8\mathbf{i} + 127\mathbf{j} - 114.7\mathbf{k}}{175.3} = 0.216\mathbf{i} + 0.724\mathbf{j} - 0.654\mathbf{k}$$
$$\therefore A_C = 175.3(0.216\mathbf{i} + 0.724\mathbf{j} - 0.654\mathbf{k}) \text{ mm} \blacklozenge$$

1.71 By inspection, a unit vector perpendicular to the door is

$$\boldsymbol{\lambda} = \sin 20^\circ\ \mathbf{i} + \cos 20^\circ\ \mathbf{j} = 0.3420\mathbf{i} + 0.9397\mathbf{j}$$

The component of **F** perpendicular to the plane of the door is

$$F_\perp = \mathbf{F} \cdot \boldsymbol{\lambda} = -22.2(0.3420) + 53.4(0.9397) = 42.6 \text{ N} \blacktriangleleft$$

1.72

For the three vectors to lie in the same plane, $\mathbf{A} \times \mathbf{B} \bullet \mathbf{C} = 0$.

$$\mathbf{A} \times \mathbf{B} \bullet \mathbf{C} = \begin{vmatrix} 2 & -1 & 2 \\ 6 & 3 & a \\ 16 & 46 & 7 \end{vmatrix} = 2(21 - 46a) + 1(42 - 16a) + 2(276 - 48) = 0$$

which gives: $540 - 108a = 0 \quad \therefore a = 5 \text{ m} \blacklozenge$

***1.73**

We first compute a unit vector $\vec{\lambda}$ that is perpendicular to plane ABC:

$$\overrightarrow{AB} = -50.8\mathbf{i} + 127\mathbf{k} \text{ mm} \qquad \overrightarrow{AC} = -50.8\mathbf{i} + 152.5\mathbf{j} \text{ mm}$$

$$\overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50.8 & 152.5 & 0 \\ -50.8 & 0 & 127 \end{vmatrix} = 19,368\mathbf{i} + 6452\mathbf{j} + 7747\mathbf{k} \text{ mm}^2$$

$$\therefore \vec{\lambda} = \frac{\overrightarrow{AC} \times \overrightarrow{AB}}{\left|\overrightarrow{AC} \times \overrightarrow{AB}\right|} = \frac{19,368\mathbf{i} + 6452\mathbf{j} + 7747\mathbf{k}}{\sqrt{19,368^2 + 6452^2 + 7747^2}} = 0.887\mathbf{i} + 0.296\mathbf{j} + 0.355\mathbf{k}$$

The normal component of F = 89**i** + 133.4**j** + 222**k** N is:

$$F_n = F \bullet \vec{\lambda} = 89(0.887) + 133.4(0.296) + 222(0.355) = 197.2 \text{ N}$$

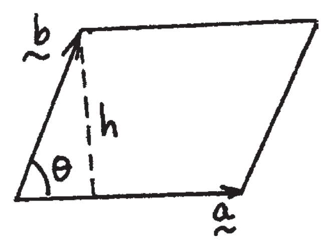
$$F_n = F_n\vec{\lambda} = 197.2(0.887\mathbf{i} + 0.296\mathbf{j} + 0.355\mathbf{k}) = 174.9\mathbf{i} + 58.4\mathbf{j} + 70\mathbf{k} \text{ N}$$

Therefore, the component of F that lies in plane ABC is:

$$F_t = F - F_n = -85.9\mathbf{i} + 75\mathbf{j} + 152\mathbf{k} \text{ N} \ \blacklozenge$$

1.74

Since a × b is perpendicular to the area,
it has the correct direction.
Check of magnitude:
$|\mathbf{a} \times \mathbf{b}| = ab\sin\theta = ah = A$ It checks!

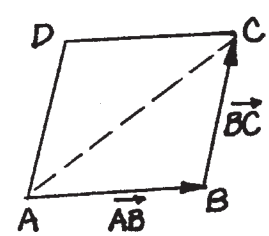


1.75

$$\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \text{area of parallelogram ABCD}$$
$$= 2(\text{area of triangle ABC})$$

$$\overrightarrow{AB} = -5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} \text{ mm} \quad \overrightarrow{BC} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} \text{ mm}$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 3 \\ 2 & -2 & 1 \end{vmatrix} = 9\mathbf{i} + 11\mathbf{j} + 4\mathbf{k} \text{ mm}^2$$



Therefore, area $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| =$
$\frac{1}{2}\sqrt{9^2 + 11^2 + 4^2} = 14.76 \text{ mm}^2$ ♦

1.76

$|\mathbf{a} \times \mathbf{b} \bullet \mathbf{c}| = |\mathbf{a} \times \mathbf{b}|\,|\mathbf{c}|\,|\cos\theta|$

From Prob. 1.74:

$|\mathbf{a} \times \mathbf{b}|$ = area of base

(shown shaded in figure)

Note that $|\mathbf{c}|\,|\cos\theta| = h$

$\therefore\ |\mathbf{a} \times \mathbf{b} \bullet \mathbf{c}|$ = (area of base) x h

= vol. of parallelpiped **Q.E.D.**