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Solution Manual for
Abstract Algebra, with Applications
Third Edition Solution Manual for

A First Course in Abstract Algebra, with Applications

Third Edition

by Joseph J. Rotman ution Manual for
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Joseph J. Rotman ¹

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1 Abstract Algebra, with Applications

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Precises for Chapter 1 Solution Manual for

See in Abstract Algebra, with Applications

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Exercises for Chapter 1

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1.1 True or false with reasons.

(i) There is a largest integer in every nonempty set

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Exercises for Chapter 1

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Exercises for Chapter 1

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There is a largest integer in every nonempty set of negative integers.

Solution. True. If C is a nonempty set

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Exercises for Chapter 1

is a largest integer in every nonempty set of negative integers.

Solution. True. If *C* is a nonempty set of negative integers, then
 $-C = \{-n : n \in C\}$

is a nonempty set of positive integers. If EXECTISES TOT CHAPTET T

alse with reasons.

There is a largest integer in every nonempty set of negative integers.
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(i) There is a largest integer in every nonempty set of negative integers.
 Solution. True. If *C* is a nonempty set of negative integers, then
 $-C = \{-n : n \in C\}$

is a nonempty set of positive i **Solution.** True. If C is a nonempty set of negati-
 $-C = \{-n : n \in C\}$

is a nonempty set of positive integers. If $-a$ is the

of $-C$, which exists by the Least Integer Axion

for all $c \in C$, so that $a \ge c$ for all $c \in C$.
 Solution. True. If C is a nonempty set of negative integers, then
 $-C = \{-n : n \in C\}$

is a nonempty set of positive integers. If $-a$ is the smallest element

of $-C$, which exists by the Least Integer Axiom, then $-a \le -c$

f $-C = \{-n : n \in C\}$
is a nonempty set of positive integers. If $-a$ is the smallest
of $-C$, which exists by the Least Integer Axiom, then $-c$
for all $c \in C$, so that $a \ge c$ for all $c \in C$.
There is a sequence of 13 consecutive $-C = \{-n : n \in C\}$

is a nonempty set of positive integers. If $-a$ is the smallest element

of $-C$, which exists by the Least Integer Axiom, then $-a \le -c$

for all $c \in C$, so that $a \ge c$ for all $c \in C$.

(ii) There is a sequen is a nonempty set of positive integers. If $-a$ is to for $-C$, which exists by the Least Integer Axi for all $c \in C$, so that $a \ge c$ for all $c \in C$.
There is a sequence of 13 consecutive natural exactly 2 primes.
Solution. Solution. False. The integers 48 through 54 are 7 consecutive
of $-C$, which exists by the Least Integer Axiom, then $-a \le -c$
for all $c \in C$, so that $a \ge c$ for all $c \in C$.
There is a sequence of 13 consecutive natural numb

or $-e$, which exists by the Exast Integer Axform, then $-a \le -e$
for all $c \in C$, so that $a \ge c$ for all $c \in C$.
There is a sequence of 13 consecutive natural numbers containing
exactly 2 primes.
Solution. True. The integ Fincte is a sequence of 15 consecutive matural natioets containing
 Solution. True. The integers 48 through 60 form such a sequence;

only 53 and 59 are primes.

There are at least two primes in any sequence of 7 consecu **Solution.** True. The integers 48 through 60 form such a sequence;
only 53 and 59 are primes.
There are at least two primes in any sequence of 7 consecutive
natural numbers.
Solution. False. The integers 48 through 54 ar

(ii) There is a sequence of 13 consecutive natural numbers containing
exactly 2 primes.
Solution. True. The integers 48 through 60 form such a sequence;
only 53 and 59 are primes.
(ii) There are at least two primes in a (iii) There are at least two primes in any sequen
natural numbers.
Solution. False. The integers 48 through 5
natural numbers, and only 53 is prime.
(iv) Of all the sequences of consecutive natural nur
2 primes, there i

Solution. The integers 48 through 05 folm start a sequence,
only 53 and 59 are primes.
There are at least two primes in any sequence of 7 consecutive
natural numbers.
Solution. False. The integers 48 through 54 are 7 cons between 48 through 54 are 7 consecutive

53 is prime.

secutive natural numbers not containing

ce of shortest length.

consisting of the lengths of such (finite)

ubset of the natural numbers.
 $\overline{81} = 9$, and 79 is no **Solution.** False. The integers 48 through
natural numbers, and only 53 is prime.
Of all the sequences of consecutive natural n
2 primes, there is a sequence of shortest len
Solution. True. The set *C* consisting of the

for the sequences of consecutive natural numbers not containing

2 primes, there is a sequence of shortest length.
 Solution. True. The set *C* consisting of the lengths of such (finite)

sequences is a nonempty subset

Solution. True. $\sqrt{79} < \sqrt{81} = 9$, and 79 is not divisible

Of all the sequences of consecutive natural numbers not containing
2 primes, there is a sequence of shortest length.
Solution. True. The set *C* consisting of the lengths of such (finite)
sequences is a nonempty subset 2 primes, there is a sequence of shortest length.
 Solution. True. The set *C* consisting of the lengths of such (finite)

sequences is a nonempty subset of the natural numbers.

79 is a prime.
 Solution. True. $\sqrt{79$

Solution. True. The set *C* consisting of the lengths of such (finite) sequences is a nonempty subset of the natural numbers.

(v) 79 is a prime.
 Solution. True. $\sqrt{79} < \sqrt{81} = 9$, and 79 is not divisible by 2, 3, 5 number.

Solution. True. We have $0 = F_0$, $1 = F_1$, $1 = F_2$, and $2 = F_3$. Use the second form of induction with base steps $n = 2$ and $n = 3$ (verifying the inductive step will show why we choose **Solution.** True. We have $0 = F_0$, $1 = F_1$, $1 = F_2$, and $2 = F_3$. Use the second form of induction with base steps $n = 2$ and $n = 3$ (verifying the inductive step will show why we choose these numbers). By the inductive h **Solution.** True. We have $0 = F_0$, $1 = F_1$, $1 = F_2$, and $2 = F_3$. Use the second form of induction with base steps $n = 2$ and $n = 3$ (verifying the inductive step will show why we choose these numbers). By the inductive h **Solution.** True. We have $0 = F_0$, $1 = F_1$, $1 = F_2$, and $2 = F_3$. Use the second form of induction with base steps $n = 2$ and $n = 3$ (verifying the inductive step will show why we choose these numbers). By the inductive **Solution.** True. We have $0 = F_0$, $1 = F_1$, $1 = F_2$, and $2 = F_3$. Use the second form of induction with base steps $n = 2$ and $n = 3$ (verifying the inductive step will show why we choose these numbers). By the inductive h **Solution.** True. We have $0 = F_0$, $1 = F_3$. Use the second form of induction wi
 $n = 3$ (verifying the inductive step wi

these numbers). By the inductive hypoth
 $n-1 \leq F_{n-1}$. Hence, $2n-3 \leq F_n$. But as

as desired.

I **Solution.** True. We have $0 = F_0$, $1 = F_1$, $1 = F_2$, and $2 = F_3$. Use the second form of induction with base steps $n = 2$ and $n - 1$ s of m inductive hypothesis, $n - 2 \le F_{n-2}$ and $n - 1 \le F_{n-1}$. Hence, $2n - 3 \le F_n$. But **Solution.** True. We have $0 = F_0$, $1 = F_1$, $1 = F_2$, and $2 = F_3$. Use the second form of induction with base steps $n = 2$ and $n = 3$ (verifying the inductive step will show why we choose these numbers). By the inductive h *n* = 5 (vertiying the matcutve step will show why we choose
these numbers). By the inductive hypothesis, $n - 2 \le F_{n-2}$ and
 $n - 1 \le F_{n-1}$. Hence, $2n - 3 \le F_n$. But $n \le 2n - 3$ for all $n \ge 3$,
as desired.
If *m* and *n* these numbers). By the matter typothesis, $n - 2 \le r_{n-2}$ and
 $n - 1 \le F_{n-1}$. Hence, $2n - 3 \le F_n$. But $n \le 2n - 3$ for all $n \ge 3$,

as desired.

If *m* and *n* are natural numbers, then $(mn)! = m!n!$.
 Solution. False. If

-
-

$$
1 + r + r2 + r3 + \dots + rn = (1 - rn+1)/(1 - r).
$$

as desired.
\n(viii) If *m* and *n* are natural numbers, then
$$
(mn)! = m!n!
$$
.
\nSolution. False. If $m = 2 = n$, then $(mn)! = 24$ and $m!n! = 4$.
\n(i) For any $n \ge 0$ and any $r \ne 1$, prove that
\n
$$
1 + r + r^2 + r^3 + \dots + r^n = (1 - r^{n+1})/(1 - r).
$$
\nSolution. We use induction on $n \ge 1$. When $n = 1$, both sides equal $1 + r$. For the inductive step, note that
\n
$$
[1 + r + r^2 + r^3 + \dots + r^n] + r^{n+1} = (1 - r^{n+1})/(1 - r) + r^{n+1}
$$
\n
$$
= \frac{1 - r^{n+1} + (1 - r)r^{n+1}}{1 - r}
$$
\n
$$
= \frac{1 - r^{n+2}}{1 - r}.
$$

\n(ii) Prove that
\n
$$
1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1.
$$

\nSolution. This is the special case of the geometric series when
\n $r = 2$; hence, the sum is $(1 - 2^{n+1})/(1 - 2) = 2^{n+1} - 1$. One can
\nalso prove this directly, by induction on $n \ge 0$.
\nwe, for all $n \ge 1$, that 10^n leaves remainder 1 after dividing by 9.
\n $\frac{1}{1 + 2 + 2^n} = \frac{1}{1 + 2^n} = \frac$

$$
1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1.
$$

 $\binom{n+1}{1-2} = 2^{n+1} - 1$. One can

 $= \frac{1 - r^{n+2}}{1 - r}.$ (ii) Prove that
 $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1.$ **Solution.** This is the special case of the geometric s
 $r = 2$; hence, the sum is $(1 - 2^{n+1})/(1 - 2) = 2^{n+1} -$

also prove this directly, by induction $=\frac{1-r^{n+2}}{1-r}$.
 $+2^2 + \cdots + 2^n = 2^{n+1} - 1$.

leave special case of the geometric series when

m is $(1-2^{n+1})/(1-2) = 2^{n+1} - 1$. One can

leaves remainder 1 after dividing by 9.

rased to say that there is an integer q_n $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.
 Solution. This is the special case of the geometric series when $r = 2$; hence, the sum is $(1 - 2^{n+1})/(1 - 2) = 2^{n+1} - 1$. One can also prove this directly, by induction on $n \ge 0$.

Show, (ii) Prove that
 $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.
 Solution. This is the special case of the geometric series when
 $r = 2$; hence, the sum is $(1 - 2^{n+1})/(1 - 2) = 2^{n+1} - 1$. One can

also prove this directly, by induction (ii) Prove that
 $1 + 2 + 2^2 + \cdots$
 Solution. This is the special c:
 $r = 2$; hence, the sum is $(1 - 2^n)$

also prove this directly, by induce

Show, for all $n \ge 1$, that 10^n leaves rema
 Solution. This may be rephra $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.
 Solution. This is the special case of the geometric series when $r = 2$; hence, the sum is $(1 - 2^{n+1})/(1 - 2) = 2^{n+1} - 1$. One can also prove this directly, by induction on $n \ge 0$.

Now, also prove this directly, by induction on $n \ge 0$.

Show, for all $n \ge 1$, that 10ⁿ leaves remainder 1 after dividing by 9.
 Solution. This may be rephrased to say that there is an integer q_n with
 $10^n = 9q_n + 1$. If 1.3 Show, for all $n \ge 1$, that 10ⁿ leaves remainder 1 after dividing l
 Solution. This may be rephrased to say that there is an integ
 $10^n = 9q_n + 1$. If we define $q_1 = 1$, then $10 = q_1 + 1$, and so th

is true.

For mainder 1 after dividing by 9.

say that there is an integer q_n with

en $10 = q_1 + 1$, and so the base step

eger q_n with
 $n + 1$
 $+ 10 = 9(10q_n + 1) + 1$.

nteger.

for all $n \ge 0$.
 $\log a^0 \le b^0$.

l so $a^0 \le b^0$. **Solution.** This may be rephrased to say that there is $10^n = 9q_n + 1$. If we define $q_1 = 1$, then $10 = q_1 + 1$, is true.
For the inductive step, there is an integer q_n with $10^{n+1} = 10 \times 10^n = 10(9q_n + 1)$
 $= 90q_n + 10 = 9($ 1 to say that there is an integer q_n with

1, then $10 = q_1 + 1$, and so the base step

m integer q_n with
 $0(9q_n + 1)$
 $0q_n + 10 = 9(10q_n + 1) + 1$.

s an integer.
 $\leq b^n$ for all $n \geq 0$.

and so $a^0 \leq b^0$.

pothesis $n^n = 9q_n + 1$. If we define $q_1 = 1$, then $10 = q_1 + 1$, and so the base step
true.
For the inductive step, there is an integer q_n with
 $10^{n+1} = 10 \times 10^n = 10(9q_n + 1)$
 $= 90q_n + 10 = 9(10q_n + 1) + 1$.
efine $q_{n+1} = 10q_n + 1$,

$$
10^{n+1} = 10 \times 10^{n} = 10(9q_n + 1)
$$

= 90q_n + 10 = 9(10q_n + 1) + 1.

 $\frac{n}{0} \leq b^n$ for all $n \geq 0$. $0 = 1 = b^0$, and so $a^0 \le b^0$. 0

 $a^n \leq b^n$.

Since *a* is positive, Theorem 1.4(i) gives $a^{n+1} = aa^n \leq ab^n$; since *b* is positive, Theorem 1.4(i) now gives $ab^n \leq bb^n = b^{n+1}$.
Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$. $n+1 = aa^n \le ab^n$; since *b* is
 $b^n = b^{n+1}$. 3
; since b is
 $\frac{1}{2}n^2 + \frac{1}{6}n$.

Since *a* is positive, Theorem 1.4(i) gives $a^{n+1} = aa^n \le ab^n$; since *b* is positive, Theorem 1.4(i) now gives $ab^n \le bb^n = b^{n+1}$.
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Solution. The Since *a* is positive, Theorem 1.4(i) gives a^{n+1} =
positive, Theorem 1.4(i) now gives $ab^n \le bb^n$ =
1.5 Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n +$
Solution. The proof is by induction on $n \ge 1$. W
1 and the right ve, Theorem 1.4(i) gives $a^{n+1} = aa^n \le ab^n$;

1.4(i) now gives $ab^n \le bb^n = b^{n+1}$.
 $2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{3}n^3 + \frac{1}{2}$

poof is by induction on $n \ge 1$. When $n = 1$, the

de is $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$.

we st $2^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$. Since *a* is positive, Theorem 1.4(i) gives $a^{n+1} = aa^n \le ab^n$; since *b* is positive, Theorem 1.4(i) now gives $ab^n \le bb^n = b^{n+1}$.
Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$.
Solution. The Since *a* is positive, Theorem 1.4(i) gives $a^{n+1} = aa^n \le$
positive, Theorem 1.4(i) now gives $ab^n \le bb^n = b^{n+1}$.
Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1) = \frac{1}{3}n$
Solution. The proof is by induction on $n \ge 1$. Wh $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1.$ $1 \equiv 1$ nce *a* is positive, Theorem 1.4(i) gives $a^{n+1} = aa^n \le$
sitive, Theorem 1.4(i) now gives $ab^n \le bb^n = b^{n+1}$.
ove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{3}n$
oution. The proof is by induction on $n \ge 1$. When $n = 1$
 Theorem 1.4(i) now gives $ab^n \leq bb^n = b^{n+1}$.

at $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{3}n^3 + \frac{1}{2}$
 a. The proof is by induction on $n \geq 1$. When $n = 1$, the right side is $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$.

e inductive Since *a* is positive, Theorem 1.4(i) gives $a^{n+1} = aa^n \le ab^n$; since *l*
positive, Theorem 1.4(i) now gives $ab^n \le bb^n = b^{n+1}$.
Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}$
Solution. The pr positive, Theorem 1.4(i) now gives $ab^n \n\t\leq bb^n$ =

1.5 Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n +$
 Solution. The proof is by induction on $n \geq 1$. W

1 and the right side is $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$.

For the i Prove that $1^2 + 2^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$.
 Solution. The proof is by induction on $n \ge 1$. When $n = 1$, the left side is 1 and the right side is $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$.

For th

Solution. The proof is by induction on
$$
n \ge 1
$$
. When $n = 1$, the left side is 1 and the right side is $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$.
\nFor the inductive step,
\n
$$
[1^2 + 2^2 + \dots + n^2] + (n+1)^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n + (n+1)^2
$$
\n
$$
= \frac{1}{3}(n+1)^3 + \frac{1}{2}(n+1)^2 + \frac{1}{6}(n+1),
$$
\nafter some elementary algebraic manipulation.
\nProve that $1^3 + 2^3 + \dots + n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$.
\n**Solution.** *Base step*: When $n = 1$, both sides equal 1.
\n*Inductive step*:
\n
$$
[1^3 + 2^3 + \dots + n^3] + (n+1)^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2 + (n+1)^3
$$
\nExpanding gives
\n
$$
\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1,
$$
\nwhich is

 $3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$. belementary algebraic manipulation.
 $3 + 2^3 + \dots + n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$.

dase step: When $n = 1$, both sides equal 1.

step:
 $3 + \dots + n^3 + (n+1)^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2 + (n+1)^3$

gives
 $\frac{1}{4}n^4 + \frac{$ after some elementary algebraic manipulation.

Prove that $1^3 + 2^3 + \cdots + n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$.
 Solution. *Base step*: When $n = 1$, both sides equal 1.
 Inductive step:
 $[1^3 + 2^3 + \cdots + n^3] + (n + 1)^3 = \frac{$

$$
[13 + 23 + \dots + n3] + (n+1)3 = \frac{1}{4}n4 + \frac{1}{2}n3 + \frac{1}{4}n2 + (n+1)3.
$$

$$
\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1,
$$

$$
\frac{1}{4}(n+1)^4 + \frac{1}{2}(n+1)^3 + \frac{1}{4}(n+1)^2.
$$

Solution. *Base step*: When $n = 1$, both sides equ
 Inductive step:
 $[1^3 + 2^3 + \cdots + n^3] + (n + 1)^3 = \frac{1}{4}n^4 + \frac{1}{2}n^5$

Expanding gives
 $\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n +$

which is
 $\frac{1}{4}(n + 1)^4 + \frac{1}{2}(n + 1)^$ $\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1,$
 $\frac{1}{4}(n + 1)^4 + \frac{1}{2}(n + 1)^3 + \frac{1}{4}(n + 1)^2.$
 $\begin{aligned}\n &\frac{4}{4}(n + 1)^4 + \frac{1}{2}(n + 1)^3 + \frac{1}{4}(n + 1)^2.\n \end{aligned}$

A $\begin{aligned}\n &\text{and}\n &\frac{4}{5} + \frac{1}{2}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n.\$ $4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n.$ *Inductive step:*
 $[1^3 + 2^3 + \cdots + n^3] + (n+1)^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2 + (n+1)^3$.

Expanding gives
 $\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1$,

which is
 $\frac{1}{4}(n+1)^4 + \frac{1}{2}(n+1)^3 + \frac{1}{4}(n+1)^2$.

Prove that $1^4 +$ $[1^3 + 2^3 + \dots + n^3] + (n+1)^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2 + (n+1)^3$.

Expanding gives
 $\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1$,

which is
 $\frac{1}{4}(n+1)^4 + \frac{1}{2}(n+1)^3 + \frac{1}{4}(n+1)^2$.

Prove that $1^4 + 2^4 + \dots + n^4 = \frac{1}{5}n^$ $\frac{1}{5} + \frac{1}{2} + \frac{1}{3} - \frac{1}{30} = 1$ as well. $[1^3 + 2^3 + \cdots + n^3] + (n+1)^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$

spanding gives
 $\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1$,

hich is
 $\frac{1}{4}(n+1)^4 + \frac{1}{2}(n+1)^3 + \frac{1}{4}(n+1)^2$.

ove that $1^4 + 2^4 + \cdots + n^4 = \frac{1}{5}n^5 + \frac{1}{2}n$ at $1^4 + 2^4 + \cdots + n^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3$
 a. The proof is by induction on $n \ge 1$. If $n - 1$

the right side is $\frac{1}{5} + \frac{1}{2} + \frac{1}{3} - \frac{1}{30} = 1$ as well.
 e inductive step,
 $f(x + 1)^4 = \frac{1}{5}n^5 + \frac{1$ which is
 $\frac{1}{4}(n+1)^4 + \frac{1}{2}(n+1)^3 + \frac{1}{4}(n+1)^2$.

Prove that $1^4 + 2^4 + \cdots + n^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$.
 Solution. The proof is by induction on $n \ge 1$. If $n - 1$, then the left side is

1, while 1.7 Prove that $1^4 + 2^4 + \cdots + n^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$.
 Solution. The proof is by induction on $n \ge 1$. If $n - 1$, then the left side is

1, while the right side is $\frac{1}{5} + \frac{1}{2} + \frac{1}{3} - \frac{1}{3$ Frove that $1 + 2 + \cdots + n = \frac{2}{5}n + \frac{2}{2}n + \frac{3}{3}n - \frac{3}{30}n$.
 Solution. The proof is by induction on $n \ge 1$. If $n - 1$, then the left

1, while the right side is $\frac{1}{5} + \frac{1}{2} + \frac{1}{3} - \frac{1}{30} = 1$ as well.

For the

Solution. The proof is by induction for
$$
n \ge 1
$$
; $1n \ne 1$, then the left side is $\frac{1}{5} + \frac{1}{2} + \frac{1}{3} - \frac{1}{30} = 1$ as well.
For the inductive step,

$$
\left[1^4 + 2^4 + \dots + n^4\right] + (n+1)^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n + (n+1)^4.
$$
It is now straightforward to check that this last expression is equal to

$$
\frac{1}{5}(n+1)^5 + \frac{1}{2}(n+1)^4 + \frac{1}{3}(n+1)^3 - \frac{1}{30}(n+1).
$$
Find a formula for $1+3+5+\dots+(2n-1)$, and use mathematical induction to prove that your formula is correct.
Solution. We prove by induction on $n \ge 1$ that the sum is n^2 .
Base Step. When $n = 1$, we interpret the left side to mean 1. Of course,
 $1^2 = 1$, and so the base step is true.
Inductive Step.
 $1+3+5+\dots+(2n-1)+(2n+1)$

$$
\frac{1}{5}(n+1)^5 + \frac{1}{2}(n+1)^4 + \frac{1}{3}(n+1)^3 - \frac{1}{30}(n+1).
$$

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 $1^2 = 1$, and so the base step is true.

$$
\left[\frac{4}{3} + 2^4 + \dots + n^4\right] + (n+1)^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n + (n+1)^4.
$$

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$$
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Inductive Step.

$$
1+3+5+\dots+(2n-1)+(2n+1)
$$

$$
= 1+3+5+\dots+(2n-1)+(2n+1)
$$

$$
= n^2 + 2n + 1
$$

$$
= (n+1)^2.
$$